STARFLEET ACADEMY

Jean-Luc Picard
Cadet, First Class
Starfleet Academy
1701 Lincoln Blvd, San Francisco, CA
www.starfleet.edu
jpicard@student.starfleet.edu

Assignment 1

COT 6410 : Spring 2325 : Dr. Vassbinder
Due Thursday, February 26, 2325

Contents

Problem 1

Define ND or $NotDominating = \{\ f \mid \text{for some } x, f(x) < x\ \}$.

(a)

Show some minimal quantification of some known primitive recursive predicate that provides an upper bound for the complexity of ND.

Answer

We can quantify with primitive recursive predicates like so:
$\exists\langle x,t\rangle[STP(f,x,t)\ \&\&\ VALUE(f,x,t) < x]$

Because of the quantifier $\exists$, the problem is no harder than a problem in the RE (or Recursively Enumerable) problem class.

(b)

Show that $K \leq_m ND$, where $K = \{\ f \mid f(f) \text{ converges }\} = \{\ f \mid \varphi_f(f)\downarrow\ \}$.

Answer

Let f be an arbitrary index of function.
Define $g_f(z) = \varphi_f(f) - \varphi_f(f) + z$
$f \in K \implies \forall z\ g_f(z) = z \implies g_f(z)\downarrow \implies g_f \in ND$
$f \notin K \implies \forall z\ g_f(z)\uparrow \implies g_f \notin ND$
Thus, $f \in K \iff g_f \in ND$
And so, $K \leq_m ND$

Problem 2

Consider the languages:
$L = \{\ a^m b^n c^t \mid t = min(m,n)\ \}$

(a)

Use the **Myhill-Nerode Theorem** to show that L **is not** a Regular Language.

Answer

Proof Idea
We need to find the equivalence classes of the language L, and show that you would need an infinite number of states to implement them. As an FSA cannot have infinite states, L cannot be regular.

Proof
$a^i b^j c^j \notin L$, where $j > i$
$a^i b^j c^j \in L$, where $i \geq j$
Because there are no limits on i, there are a countably infinite number of these classes. Because a FSA must have a finite number of states, you cannot construct one to recognize L. Therefore L is not a Regular Language.

Problem 2 continued on next page. . .

(b)

Use the **Pumping Lemma for CFLs** to show that L **is not** a Context Free Language.

Answer

This is a proof by contradiction.
Assume L is a Context Free Language.
And the Pumping Lemma for CFL states that any string $s \in L = uvxyz$, where:
1. $\forall i \geq 0,\ uv^ixy^iz \in L$
2. $\mid vy \mid > 0$
3. $\mid vxy \mid \leq p$

There are 2 cases to consider: (1) $t = n$, and (2) $t = m$.

Case 1
Choose string $s = a^m b^n c^t$, where $t = n$.
Then there are more a than b, for example $a^3b^2c^2 = aaabbcc$.
Let i be 0.
If vxy contains both a and c, then with $i = 0$, the number of c are not equal to either a or b.
(for example $a^3b^2c^2 = aaabbcc = aaa^0bbc^0c = aabbc \ \notin \ L$)
Thus we have a contradiction.

Case 2
Choose string $s = a^m b^n c^t$, where $t = m$.
Then there are less a than b, for example $a^2b^3c^2 = aabbbcc$.
Let i be 0.
If vxy contains a and b but no c, then with $i = 0$, $t \neq m$ and the number of $c \neq \min(m, n)$.
(for example $a^2b^3c^2 = aabbbcc = aa^0bbb^0cc = abbcc \ \notin \ L$)
Thus we have a contradiction.

Therefore L cannot be a Context Free Language.

Problem 3

Explain what a tachyon is.

Answer

A tachyon is a subatomic particle that naturally exists at faster-than-light velocities. They can often be associated with time travel or produced as a byproduct of temporal distortions. Tachyons can exist both naturally, as in the tachyon eddies in the Bajoran system, and as the byproducts of certain technologies, such as cloaking devices and transporters. The detection of tachyons may lead to clues about recent temporal or spactial anomalous.