INSERT YOUR TITLE HERE

BY

NAME SURNAME, B.S., M.S.

A dissertation submitted to the Graduate School

in partial fulfillment of the requirements

for the degree

Doctor of Philosophy

Major Subject: Mathematics

New Mexico State University

Las Cruces New Mexico

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"Thesis Title Goes Here," a dissertation prepared by Name Surname in partial fulfillment of the requirements for the degree, Doctor of Philosophy, has been approved and accepted by the following:

Name I. Surname
Interim Dean of the Graduate School

Name I. Surname
Chair of the Examining Committee

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DEDICATION

I dedicate this work to my cat Tom, my mouse Jerry, and Pistol Pete.

ACKNOWLEDGMENTS

I would like to thank my advisor, Name I. Surname, for his encouragement, interest, and patience. Personally, I would like to thank him for sharing his knowledge which has enriched my study in topology.

VITA

January 1, 1970	Born at City Name, State Name, Country Name
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PROFESSIONAL AND HONORARY SOCIETIES

American Mathematical Society

PUBLICATIONS [or Papers Presented]

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FIELD OF STUDY

Major Field: Algebraic Topology

ABSTRACT

INSERT YOUR TITLE

HERE

BY

NAME SURNAME, B.S., M.S.

Doctor of Philosophy

New Mexico State University

Las Cruces, New Mexico, 2022

Dr. Name I. Surname , Position

In this work we present... (abstract goes here)

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1 INTRODUCTION

This document has been prepared to serve as a template for preparation of a master's or Ph.D. thesis in mathematics at New Mexico State University. The template uses the LaTeX 2_ε document preparation system as the basic tool. Before you attempt to use the template, you should become familiar with the LaTeX 2_ε system. For example, you may browse the directory `usr/local/teTeX/texmf/doc/latex` on the Department of Mathematical Sciences UNIX server for essential information about LaTeX 2_ε. You may also wish to invest in a copy of Leslie Lamport's user guide and reference manual [3].

However, the LaTeX 2_ε document preparation system by itself is not sufficient for preparing an NMSU thesis. On one hand, some of the formatting requirements of the NMSU graduate school conflict with the default formatting provided by the document class `article` provided by the standard LaTeX 2_ε package. Some of the NMSU formatting requirements may be fulfilled by settings made in the preamble of your LaTeX document. To handle other parts of the graduate school's formatting guidelines, we have provided a new document class `nmsuth01` that seems to solve most of these local formatting issues. You will need to have the file `nmsuth01.cls` in the directory with the files comprising your thesis in order for LaTeX to find it. Note that when the file `nmsuth01.cls` loads, LaTeX reports a warning that an unusual document class is in use. This is normal and should not effect the creation of the `.dvi` file. Finally, we should warn potential users that we have constructed the file `nmsuth01.cls` to handle issues that seem to be essential to us. It is possible that "extreme" users will have formatting problems that necessitate additional modifications to the article class file.

After you have gained some experience with LaTeX and with literature in your speciality, it will become clear that certain demands placed on the typesetter by higher mathematics cannot be fulfilled by the LaTeX 2_ε package itself. To get around these problems, users of TeX have developed developed special packages to make the typesetting of mathematics easier. The American Mathematical Society has a package of macros to handle such issues as formatting for statements of theorems, definitions, examples, remarks, and so on. Other parts of the American Mathematic Society package deal with complicated display alignment and line breaking issues. Other packages have been developed to handle inclusion of graphics into mathematical documents. All of the packages used in the template are installed on the Department of Mathematics UNIX server. For general information about what TeX features are currently available browse the directory `usr/local/teTeX/texmf/doc` on the UNIX server. Your thesis advisor should also be willing to draw your attention to specific packages you might find especially useful.

This files for this template were prepared by Pat Morandi and Ross Staffeldt. All the `.tex` files for the template were prepared using the emacs editor.

Before you start creating files and problems for yourself, you should acquire some basic background information about writing in the mathematical sciences. A basic reference is Nicholas J. Higham's book [2], which includes discussion of what differentiates good writing from poor writing, preparation of talks and posters, as well as preparation of theses and papers. He also has material on TeX systems and a brief introduction to the emacs editor. The American Mathematical Society publication [7] has general information about mathematical typesetting which is still useful, although the book predates the TeX-age. We conclude with a brief description of the contents of this template.

Chapter 2 is an extract from the 1997 Ph.D. thesis of Eduardo Quiñones-Rico. It is included to illustrate the some of the features made available in the packages `amsmath` and `amsthm` which are in the macro package AMS-LaTeX distributed free of charge by the American Mathematical Society. Among the features illustrated in this section are the environments for theorems, proofs, lemmas, remarks, and examples of "cases" constructions. This section also illustrates some commutative diagrams illustrating the package `xypic`. In later versions of the template we hope to include examples from other NMSU theses illustrating problems and solutions associated to communicating other types of mathematics.

Chapter 3 provides a sample table containing mathematical entries. The table is also extracted from Eduardo's thesis. There are no special packages called for in this section. The main purpose of the section is to give a reason for creating a list of tables following the table of contents page.

Chapter 4 provides two figures which are the realizations of two encapsulated PostScript files. Creating and incorporating graphics is rather sophisticated business. For detailed information on the incorporation of graphics into a LaTeX 2_ε document go to some directory under your UNIX account and use the command

```
cp /usr/local/teTeX/texmf/doc/latex/graphics/epslatex.ps epslatex.ps
```

to transfer a copy of the document "Using EPS Graphics in LaTeX 2_ε Documents" to your account, where you can view it or print it for later reference.

2 HOMOLOGY

We now introduce the concepts of simplicial module and of the homology associated to a simplicial module. As a variation of this, we recall the definition of cyclic module and its cyclic homology first given by Loday [4].

The letter k will always denote a commutative ring with identity.

2.1 Simplicial Homology of a Simplicial Module

We define simplicial modules and their homology. Also, two examples are given. The first example is the free simplicial module associated to a simplicial set. Our second example is the Hochschild complex of a unitary algebra A over a ring k.

Definition 2.1. A simplicial k-module M_* is a family of k-modules $\{M_n\}_{n\geq 0}$ together with k-module homomophisms $d_i\colon M_n \to M_{n-1}$ and $s_i\colon M_n \to M_{n+1}$ for $0 \leq i \leq n$ satisfying the following identities:

$$d_i d_j = d_{j-1} d_i \quad \text{for} \quad i < j. \tag{1}$$

$$s_i s_j = s_{j+1} s_i \quad \text{for} \quad i \leq j \tag{2}$$

$$d_i s_j = \begin{cases} s_{j-1} d_i & \text{if } i < j, \\ id & \text{if } i = j \text{ or } i = j+1, \\ s_j d_{i-1} & \text{if } i > j+1. \end{cases} \tag{3}$$

In order to define the homology of a simplicial module define $b\colon M_n \to M_{n-1}$ to be the boundary map such that

$$b = \sum_{i=0}^{n} (-1)^i d_i$$

Relations (1) imply $b^2 = 0$, so we obtain a chain complex (M_*, b) and we have the following definition:

Definition 2.2. Let M_* be a simplicial k-module. The homology of the simplicial module M_* is given by

$$H_n(M_*) = H_n(M_*, b)$$

where $H_n(M_*, b)$ is the n-th homology of the complex (M_*, b).

Example 2.3. A simplicial set X is a family of sets $\{X_n\}_{n\geq 0}$ together with functions d_i and s_i satisfying relations (1)-(3). A simplicial set gives rise to a free simplicial k-module denoted $k[X]$ by taking $k[X]_n = k[X_n]$, the free k-module with basis X_n. In this case we usually write $H_*(k[X]) = H_*(X; k)$. Let Y be a topological space and let $X = \mathrm{Sin}(Y)$ be the singular complex of Y. This has

$$X_n = \{\sigma\colon \Delta^n \to Y : \sigma \text{ is continuous}\}$$

with faces and degeneracies induced by the cofaces and codegeneracies of Δ^n, where Δ^n is the geometric simplex of dimension n. Then $H_*(X; k)$ is singular homology of the space Y with coefficients in k.

Example 2.4. Let A be an associative k-algebra and let A_* be the simplicial k-module defined by

$$A_n = A^{\otimes(n+1)} = A \otimes_k A \otimes_k \cdots \otimes_k A \qquad (n+1) \text{ factors}$$

A generator of $A^{\otimes(n+1)}$ is denoted by $(a_0, \ldots a_n)$. Let

$$d_i\colon A_n \to A_{n-1} \quad \text{and} \quad s_i\colon A_n \to A_{n+1}$$

be such that

$$d_i(a_0,\dots,a_n) = \begin{cases} (a_0,\dots,a_ia_{i+1},\dots a_n) & \text{if } 0 \le i < n, \\ (a_na_0,a_1,\dots,a_{n-1}) & \text{if } i = n. \end{cases}$$

$$s_i(a_0,\dots,a_n) = (a_0,\dots,a_i,1,a_{i+1},\dots,a_n) \quad \text{for} \quad 0 \le i \le n$$

It can be verified that these functions satisfy the simplicial identities and, therefore, the family of modules $A_n = A^{\otimes(n+1)}$ together with these maps d_i and s_i is a simplicial module. Its homology is known as the Hochschild homology of the algebra A and is denoted $HH_*(A)$.

2.2 Cyclic Homology of a Cyclic Module

In this section we describe the definition of cyclic homology given by Loday and Quillen [4].

Definition 2.5. A cyclic module M is a simplicial module together with an action of the cyclic group $C_{n+1} = \langle t_{n+1} : (t_{n+1})^{n+1} = 1\rangle$ on M_n, for each n, such that the following additional relations hold

$$\begin{aligned} d_it_{n+1} &= t_nd_{i-1} \quad \text{and} \quad s_it_{n+1} = t_{n+2}s_{i-1} \quad \text{for} \quad 1 \le i \le n, \\ d_0t_{n+1} &= d_n \qquad \text{and} \quad s_0t_{n+1} = t_{n+2}^2s_n \end{aligned} \tag{4}$$

where $d_i\colon M_n \to M_{n-1}$ and $s_i\colon M_n \to M_{n+1}$ for $i = 0,1,\dots,n$ are the simplicial structure maps.

Next we will define the cyclic homology of a cyclic module. The bicomplex we describe now will reappear in a more conceptual way, after we develop Tor functors associated to cyclic and cocyclic modules.

If we define two operators T_{n+1} and N_{n+1} by

$$T_{n+1} = (-1)^n t_{n+1} \quad \text{and} \quad N_{n+1} = 1 + T_{n+1} + \cdots + T_{n+1}^n$$

then we have the following lemma:

Lemma 2.6. *Let $\mathcal{C}(M)$ be the diagram of modules and homomorphisms*

$$\begin{array}{ccccccccc}
\vdots & & \vdots & & \vdots & & \vdots & & \\
b\big\downarrow & & b'\big\downarrow & & b\big\downarrow & & b'\big\downarrow & & \\
M_2 & \xleftarrow{1-T_3} & M_2 & \xleftarrow{N_3} & M_2 & \xleftarrow{1-T_3} & M_2 & \xleftarrow{N_3} & \cdots \\
b\big\downarrow & & b'\big\downarrow & & b\big\downarrow & & b'\big\downarrow & & \\
M_1 & \xleftarrow{1-T_2} & M_1 & \xleftarrow{N_2} & M_1 & \xleftarrow{1-T_2} & M_1 & \xleftarrow{N_2} & \cdots \\
b\big\downarrow & & b'\big\downarrow & & b\big\downarrow & & b'\big\downarrow & & \\
M_0 & \xleftarrow{1-T_1} & M_0 & \xleftarrow{N_1} & M_0 & \xleftarrow{1-T_1} & M_0 & \xleftarrow{N_1} & \cdots
\end{array} \tag{5}$$

where $b, b' \colon M_n \to M_{n-1}$ are defined by

$$b = \sum_{i=0}^{n} (-1)^i d_i \quad \textit{and} \quad b' = \sum_{i=0}^{n-1} (-1)^i d_i$$

Then the squares anticommute, the rows and columns are complexes, so $\mathcal{C}(M)$ is a bicomplex.

Proof. See [4, page 52] □

Definition 2.7. Let M_* be a cyclic module. The cyclic homology of M_* is denoted $HC_*(M_*)$ and is defined as

$$HC_*(M_*) = H_*(\mathrm{Tot}(\mathcal{C}(M)))$$

where $\mathcal{C}(M)$ is the the bicomplex of lemma 2.6.

Example 2.8. Let A_* be the simplicial module defined in example 2.4. The action of the cyclic group $C_{n+1} = \langle t_n : (t_{n+1})^{n+1} = 1 \rangle$ on A_n is given by:

$$t_{n+1}(a_0, \ldots, a_n) = (a_n, a_1, \ldots, a_{n-1})$$

It can be verified that under these definitions A_* becomes a cyclic module. In this case we write $HC_*(A)$ instead of $HC_*(A_*)$ for the cyclic homology of A.

The vertical complexes (M_*, b') in the bicomplex (5) are contractible, with contracting homotopy $h = T_{n+1}s_n \colon M_n \to M_{n+1}$. Loday's Killing Contractible Complexes Lemma [4, page 55] allows us to delete all of these columns of the bicomplex (5) obtaining the following double complex:

$$\begin{array}{ccccc} \vdots & & \vdots & & \vdots \\ \downarrow b & & \downarrow b & & \downarrow b \\ M_2 & \xleftarrow{B} & M_1 & \xleftarrow{B} & M_0 \\ \downarrow b & & \downarrow b & & \\ M_1 & \xleftarrow{B} & M_0 & & \\ \downarrow b & & & & \\ M_0 & & & & \end{array}$$

Here $B = (-1)^{n+1}(1 - t_{n+1})hN \colon M_n \to M_{n+1}$ and b is the Hochschild boundary map. This bicomplex is called the $\mathcal{B}M$ complex or $(B$-$b)$-complex. For the particular case of the cyclic module associated to the algebra $A = T(V)$ as described in 2.4, the $(B$-$b)$-complex is used to give an explicit computation [5]. The result is given in the following theorem.

Theorem 2.9. *Let V be a module over k, and let $T(V)$ be its tensor algebra. Then the cyclic homology $HC_*(T(V))$ is given by:*

$$HC_*(T(V)) = HC_*(BS^1, k) \oplus \sum_{m \geq 1} H_*(C_m, V^{\otimes m})$$

where the generator t_m of C_m acts on $V^{\otimes m}$ as

$$t_m(v_1 v_2 \cdots v_m) = v_m v_1 \cdots v_{m-1}$$

Proof. See [5, page 581]. □

3 A SAMPLE OF TEXT WITH TABLE

Now we want to describe a tricomplex $B\mathcal{D}^3$ so that its total differential corresponds to the total differential d'' in the version of $\mathrm{Tot}(\mathcal{D}^3)$ modified by deleting all the vertical b'-complexes from the tricomplex $\mathcal{D}^3$. Let us consider the tricomplex $B\mathcal{D}^3$ as a tridimensional array that results after replacing all the vertical b'-complexes by zero complexes. The rules determining the maps connecting these modules are described in the following table.

Table 1: Table of Maps

Case	To determine	Look at	In
(1)	$(B\mathcal{D}^3)_{p,q,0} \to (B\mathcal{D}^3)_{p+q-j-1,j,0}$ $0 \leq j \leq p+q-1$	$(j{+}1, q{+}1)$-entry of $B_{p+q,p+q+1}$	d'_{p+q}
(2)	$(B\mathcal{D}^3)_{p,q,0} \to (B\mathcal{D}^3)_{p+q-j-2,j,1}$	$(j{+}1, q{+}1)$-entry of $B_{p+q-1,p+q+1}$	d'_{p+q}
(3)	$(B\mathcal{D}^3)_{p,q,1} \to (B\mathcal{D}^3)_{p+q-j,j,0}$	$(j{+}1, q{+}1)$-entry of $B_{p+q+1,p+q+1}$	d'_{p+q+1}
(4)	$(B\mathcal{D}^3)_{p,q,1} \to (B\mathcal{D}^3)_{p+q-j-1,j,1}$	$(j{+}2, q{+}1)$-entry of $B_{p+q,p+q+1}$	d'_{p+q+1}

Let us start with case (1) in Table 1.

4 SAMPLE TEXT WITH GRAPHICS

Sometimes you may want to include in your LaTeX document graphical output generated by a program like Maple, Mathematica, or Matlab. There is a LaTeX2$_\varepsilon$ graphics bundle which can be loaded by specifying `\usepackage{graphicx}` in the preamble of the document. Documentation of the package is supplied by the preprint [6]. Instructions for obtaining a copy of this document are found in section 1. The graph in Figure 1 is from a PostScript file

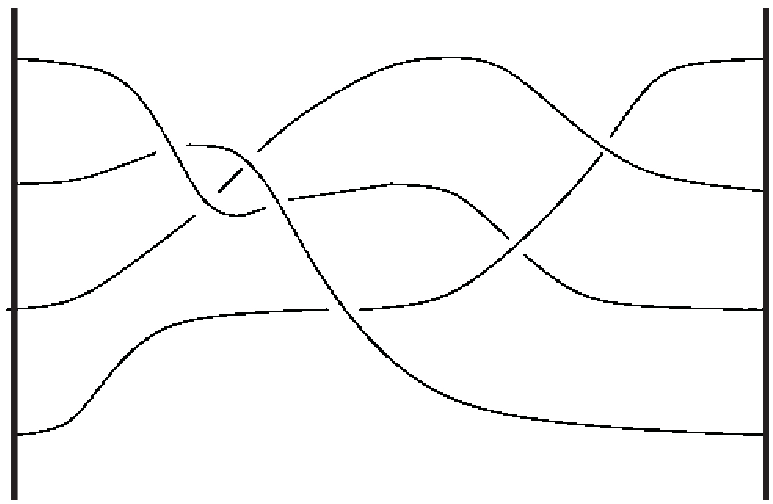

Figure 1: This is an inserted EPS graphic

generated by Maple and then converted to an Encapsulated Postcript File by the `ps2epsi` utility in ghostscript.

The figure is placed in the file `graphics.tex` immediately after the sentence ending "also generated by Maple." Obviously it will not fit on the page with that sentence, so LaTeX places the figure at the top of the next page. In the original version of the template, the next page was the start of the list of bibliographic references. In order to get the list of references to start on a fresh page, we have had to add this text to the file. The moral of the story is that typesetting decisions can have unintended consequences.

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