Problem 1. Show that there exists no nontrivial unramified extensions of $\mathbb{Q}$.

Solution: If $K/\mathbb{Q}$ is a nontrivial number field, then $|\operatorname{disc} K| > 1$. But then $\operatorname{disc} K$ has a prime factor so that some prime ramifies in K. □

Problem 2. Complete the following:

(a) How does one prove a cotheorem?

(b) Compute $\int \cos x \, dx$.

(c) How does one square $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$?

Solution:

(a) Use rollaries.

(b) We have

$$\int \cos x \, dx = \sin x + C \tag{1}$$

We can check (1):

$$\frac{d}{dx}(\sin x + C) = \cos x$$

(c) This is routine.

□

Problem 3. Prove that $\sqrt{2}$ is irrational.

Proof. Assume that $\sqrt{2} = \frac{a}{b}$, where $a, b \in \mathbb{Z}$. Without loss of generality, we may assume $\gcd(a, b) = 1$. Then we have

$$\begin{aligned} \sqrt{2} &= \frac{a}{b} \\ \sqrt{2}^2 &= \left(\frac{a}{b}\right)^2 \quad (2) \\ 2 &= \frac{a^2}{b^2} \\ a^2 &= 2b^2 \quad (3) \end{aligned}$$

But then from (3), we know that a^2 is even so that a is even. But then we must have

$$2a^2 = b^2$$

so that b^2 is even, implying b is even. But then $\gcd(a, b) \geq 2$, a contradiction. □