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Term Project

Price Dynamics in Financial Markets : A Kinetic Approach

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February 16, 2017

Abstract

Econophysics is an interdisciplinary research field, applying theories and methods originally developed by physicists in order to solve problems in economics, usually those including uncertainty or stochastic processes and nonlinear dynamics. Some of its application to the study of financial markets has also been termed statistical finance referring to its roots in statistical physics.

The recent events of the 2008 world's financial crisis and its uncontrolled effect propagated among the global economic system, has produced a deep rethink of some paradigm and fundamentals in economic modeling of financial markets.

Any reasonable model need to rely on some fundamental hypotheses and to rest on a theoretical framework, which should be able to provide some basic and universal principles, this is the way all the models arising from the physical world are build up.

One of the most classical approach has been to consider the efficient market hypothesis.. It relies on the belief that securities markets are extremely efficient in reflecting information about individual stocks and about the stock market as a whole. When information arises, the news spread very quickly and are incorporated into the prices of securities without delay. Thus, neither technical analysis, which is the study of past stock prices in an attempt to predict future prices, nor even fundamental analysis, which is the analysis of financial information such as company earnings, asset values, etc., to help investors select undervalued stocks, would enable an investor to achieve returns greater than those that could be obtained by holding a randomly selected portfolio of individual stocks with comparable risk.

We derive a set of equations which are a simple model for investor behavior in a theoretical financial market. The model incorporates the emotional aspect of investor sentiment with memory of price history which decays exponentially in time. Within this model, the emotional reaction of the body of investors is to buy when the recent price has been increasing and sell when it has been decreasing. The rational motivations are based on capitalizing on the difference between the price and intrinsic value, with the possibility of some inertia in taking action. These two competing effects provide the basis for fluctuations and instability.

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Chapter 1

PRICE DYNAMICS IN FINANCIAL MARKETS

1.1 Efficient Market Hypothesis

The efficient market hypothesis is associated with the idea of a random walk,which is widely used in the finance literature to characterize a price series where all subsequent price changes represent random departures from previous prices. The logic of the random walk idea is that if any information is immediately reflected in stock prices, then tomorrow's price change will reflect only tomorrow's news and will be independent of the price changes today. Thus, resulting price changes must be unpredictable and random.

Strongly linked to the market efficiency hypothesis, is the assumption of rational behavior among the traders. Rationality of traders can be basically reassumed in two main features. First, when they receive new information, agents update their beliefs by evaluating the probability of hypotheses accordingly to Bayes' law. Second, given their beliefs, agents make choices that are completely rational, in the sense that they arise from an optimization process of opportune subjective utility functions.

1.2 Behavioral Finance Theory

By the beginning of the twenty-first century, the intellectual dominance of the efficient market hypothesis had become far less universal. Many financial economists and statisticians began to believe that stock prices are at least partially predictable. A new breed of economists emphasized psychological and behavioral elements of stock-price determination. The behavioral finance approach has emerged in response to the difficulties faced by the traditional paradigm.

It relies in the fact that some financial phenomena can be better understood using models in which some agents are not fully rational. In some behavioral finance models, agents fail to update their beliefs correctly. In other models, agents apply Bayes' law properly but make choices that are questionable, in the sense that they are incompatible with the optimization of suitable utility functions.

1.3 Prospect Theory

A strong impact in the field of behavioral finance has been given by the introduction of the prospect theory by Kahneman and Tversky. They present a critique of expected utility theory as a descriptive model of decision making under risk and develop an alternative model. Under prospect theory, value is assigned to gains and losses rather than to final assets and probabilities are replaced by decision weights.The theory which they confirmed by experiments predicts a distinctive fourfold pattern of risk attitudes: risk aversion for gains of moderate to high probability and losses of low probability, and risk seeking for gains of low probability and losses of moderate to high probability.

Further development in this direction, led to the discovery that that people systematically overreacting to unexpected and dramatic news events results in substantial weak-form inefficiencies in the stock market.

1.4 Agent Based Modeling

Recently, these methods have given an important contribute and provided a huge quantity of numerical simulations.The idea is to produce a big mass of artificial data and to observe how they can fit with empirical observations. This approach is now also supported by the availability of many recorded empirical data.The aim of the construction of such microscopic models of financial markets is to reproduce the observed statistical features of market movements (e.g. fat tailed return distributions, clustered volatility, cycles, crashes) by employing highly simplified models with large numbers of agents. Microscopic models of financial markets are highly idealized as compared to what they are meant to model.The relevant part of physics that is used to build such models of financial markets consists in methods from statistical mechanics. This attempt by physicists to map out the statistical properties of financial markets considered as complex systems is usually referred to as econophysics.

The need to recover mathematical models which can display such scaling properties, but also capable to deal with systems of many interacting agents and to take into account the effects of collective endogenous dynamics, put the question on the choices of the most appropriate mathematical framework to use.

Besides numerical simulations, it is of paramount importance to have a rigorous mathematical theory which permits to identify the essential features in the modelling originating the stylized facts. The classical framework of stochastic differential equations which played a major rule in financial mathematics seems inadequate to describe the dynamics of such systems of interacting agents and their emerging collective behavior.

Chapter 2

KINETIC MODELLING FOR PRICE FORMATION

2.1 Introduction

Kinetic theory was introduced in order to give a statistical description of systems with many interacting particles. Rarefied gases can be thought as a paradigm of such complex systems, in which particles are described by random variables which represents their physical states, like position and velocity. A Boltzmann equation then prescribes the time evolution for the particles density probability function.

The above theoretical model seems to fit very well with the necessity to prescribe how the trading agents interacting in a stock market are leaded to form their expectations and re-evaluate their choices on the basis of the influence placed on the neighbor agents' behavior rather than the flux of news coming from some fundamental analysis or direct observations of the market dynamic.The kinetic approach reveals particularly powerful when from some simple local interaction rules some global features for the whole system has to be derived, but also in the study of asymptotic regimes and universal behaviors described by Fokker-Planck equations.

2.2 Opinion Modeling

The collective behavior of a system of trading agents can be described by introducing a state variable y ϵ [-1,1] and the relative density probability function f(y) which, for each agent, represent respectively the propensity to invest and the probability to be in such a state. Positive values of y represent potential buyers, while negative values characterize potential sellers, close to y = 0 we have undecided agents. Clearly,

$$\rho(t) = \int_{-1}^{1} f(y,t)dy \tag{2.1}$$

Equation 2.1 represents the number density. Moreover we define the mean investment propensity by equation 2.2,

$$Y(t) = \frac{1}{\rho(t)} \int_{-1}^{1} f(y,t)ydy \tag{2.2}$$

Traders are allowed to compare their strategies and to re-evaluate them on the basis of a compromise opinion dynamic. This is done by assigning simple binary interaction rules, where, if the pair (y,y_*) and (y',y'_*) represents respectively the pre-interaction and the post-interaction opinions, we have:

$$y' = (1 - \alpha_1(H(y))y + \alpha_1 H(y)y_* + D(y)\eta \tag{2.3}$$

$$y'_* = (1 - \alpha_1(H(y_*))y_* + \alpha_1 H(y_*)y_* + D(y)\eta_* \tag{2.4}$$

Here $\alpha_1 \epsilon$ [0,1] measures the importance the individuals place on others opinions in forming expectations about future price changes.The random variables η and η_* are assumed are assumed distributed accordingly to $\theta(\eta)$ with zero mean and variance σ^2 and measure individual deviations to the average behavior. The functions H(y) and D(y) characterize respectively, herding and diffusive behavior. Simple examples of herding function and diffusion function are given by:

$$H(y) = a + b(1- \mid y \mid) \tag{2.5}$$

$$D(y) = (1 - y^2)^2 \gamma \tag{2.6}$$

With $0 \leq$ a + b ≤ 1, a ≥ 0, b > 0, $\gamma > 0$.

2.3 Market Influence

The traders are also influenced by the dynamics of stock market's price, so a coupling with the price dynamic has to be considered. With the same kinetic setting we define the probability density V(s,t) of a given price s at time t. The market price S(t) is then defined as the mean value.

$$S(t) = \int_0^\infty V(s,t)sds \tag{2.7}$$

Price changes are modeled as endogenous responses of the market to imbalances between demand and supply characterized by the mean investment propensity accordingly to the following price adjustment.

$$s' = s + \beta\rho Y(t)s + \eta S \tag{2.8}$$

Where, $\beta > 0$ represents the price speed evaluation and η is a random variable with zero mean and variance ζ^2 distributed accordingly to $\Psi(\eta)$.

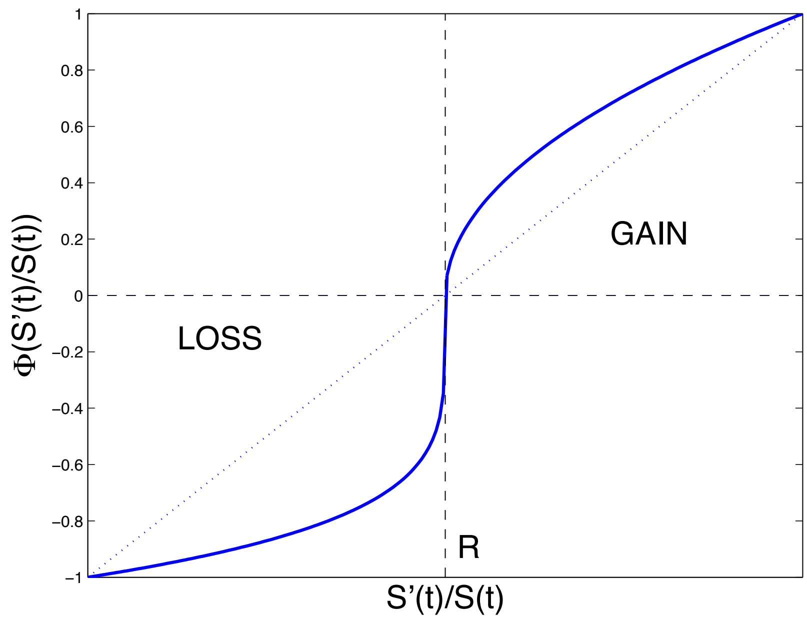

Figure 2.1: Loss vs. Gain

To take into account the influence of the price in the mechanism of opinion formation of traders, we introduce a normalized value function $\Theta = \Theta(\frac{\frac{dS(t)}{dt}}{S(t)})$ in [-1,1] in the sense of Kahneman and Tversky that models the reaction of individuals towards potential gain and losses in the market. Thus we reformulate the binary interaction rules in the following way:

$$y' = (1 - \alpha_1(H(y) - \alpha_2)y + \alpha_1 H(y)y_* + \alpha_2\Theta(y)\eta \tag{2.9}$$

$$y'_* = (1 - \alpha_1(H(y_*) - \alpha_2)y_* + \alpha_1 H(y_*)y_* + \alpha_2\Theta(y)\eta_* \tag{2.10}$$

Here, $\alpha_1\epsilon$ [0,1] and $\alpha_2\epsilon$[0,1] with $\alpha_1 + \alpha_2 \leq 1$, measure the importance the individuals place on others opinions and actual price trend in forming expectations about future price changes. This permits to introduce behavioral aspects in the market dynamic and to take into account the influence of psychology and emotivity on the behavior of the trading agents.

Note: Except for the particular shape of the value function, if the mean propensity is initially (sufficiently) positive then it will continue to grow together with the price and the opposite occurs if it is initially (sufficiently) negative. The market goes towards a boom (exponential grow of the price) or a crash (exponential decay of the price).

2.4 Lognormal Behavior

A set of Boltzmann equations for the evolution of the unknown densities f(y,t) and V(s,t) can be obtained using the standard tools of kinetic theory. Such systems are defined by the equations 2.11 and 2.12.

$$\frac{\partial f}{\partial t} = Q(f, f) \tag{2.11}$$

$$\frac{\partial V}{\partial t} = L(V) \tag{2.12}$$

where the quadratic operator Q and the linear operator L can be conveniently defined in weak form by equation 2.13 and 2.14.

$$\int_{-1}^{1} Q(f, f)\Phi(y)dy = \int_{[-1,1]^2} \int_{R^2} B(y, y_*)f(y)f(y_*)(\Phi(y) - \Phi(y'))d\eta d\eta_* dy dy_* \tag{2.13}$$

$$\int_{0}^{\infty} L(V)\Phi(s)ds = \int_{0}^{\infty} \int_{R} b(s)V(s)(\Phi(s') - \Phi(s))d\eta ds \tag{2.14}$$

In the above equations Φ is a test function and the transition rates have the form given by the equations 2.15 and 2.16.

$$B(y, y_*) = \Theta(\eta)\Theta(\eta_*)\chi(\mid y' \mid \leq 1)\chi(\mid y'_* \mid \leq 1) \tag{2.15}$$

$$b(S) = \Psi(\eta)\chi(s' \geq 0) \tag{2.16}$$

With $\chi(.)$ representing the indicator function.

A simplified Fokker-Planck model which preserves the main features of the original Boltzmann model is obtained under a suitable scaling of the system. In such scaling all agents interact simultaneously with very small variations of their investment propensity.This allows us to recover the following Fokker-Plank system:

$$\frac{\partial f}{\partial t}\frac{\partial[(\rho\alpha_1 H(y)(Y - y) + \rho\alpha_2(\Theta - y))f]}{\partial y} = \frac{\sigma^2\rho}{2}\frac{\partial^2[D^2(y)f]}{\partial^2 y^2} \tag{2.17}$$

$$\frac{\partial V}{\partial t} + \frac{\partial[\beta\rho Y s V]}{\partial s} = \frac{\zeta^2}{2}\frac{\partial^2 s^2 V}{\partial^2 s^2} \tag{2.18}$$

where we kept the original notations for all the scaled quantities.

The above equation for the price admits the self similar lognormal solution.

$$V(s, t) = \frac{1}{s(2log(Z(t)^2)\pi)^{\frac{1}{2}}} exp(\frac{-log(sZ(t))^2}{2log(Z(t)^2)}) \tag{2.19}$$

Where, $Z(t) = \sqrt{E(t)}/S(t)$, and E(t) satisfies the differential equation given by equation 2.20.

$$\frac{dE}{dt} = (2\beta Y + \zeta^2)E \tag{2.20}$$

2.5 A Different Strategy

We consider now in the stock market the presence of traders who deviate their strategy from the mass. We introduce trading agents who rely in a fundamental value for the traded security. They are buyer while the price is below the fundamental value and seller while the price is above. Expected gains or losses are then evaluated from deviations of the actual market price and just realized only wether or not the price will revert towards the fundamental value. Such agents are not influenced by other agents' opinions.

The microscopic interactions rules for the price formation now reads.

$$s' = s + \beta(\rho Y(t)s + \rho_F \gamma(S_F - s)) + \eta_s \tag{2.21}$$

Where, S_F represents the fundamental value of price, ρ_F is the number density for such trading agents performing a different strategy, while γ is the reaction strength to the deviations from fundamental value. If we are interested in steady states we can ignore the possibility of a strategy exchange between traders and the resulting kinetic system has the same structure (2.17 and 2.18).

Chapter 3

MARKET FLUCTUATION PSYCHOLOGY

3.1 Introduction

Large price fluctuations in financial markets are often far in excess of the changes in intrinsic value. These fluctuations are generally attributed to the psychological reactions and overreactions to rapid but smaller changes in fundamental value. In this paper, we model the behavior of a theoretical financial market based on the psychological motivation of a body of investors.

The modeling of a financial system with a large number of decision makers is analogous to modeling a physical system consisting of many degrees of freedom. The alternatives can be grouped into at least two categories:

1. The states in which the investor is in, for example, he is in a buying mode or a selling mode is determined by giving individual probabilistic weighing. Within this model, the emotional reaction of the body of investors is to buy when the recent price has been increasing and sell when it has been decreasing.This is analogous to a statistical mechanical approach to thermodynamics.
2. In this model one considers the total number of investors in each state and the total rate of transition of these investors from one state to another just like the transfer of molecules from one state to another. This is analogous to chemical kinetics or Boltzmann dynamics for a gas.

The intent of this section is that to show that such price fluctuations can be modeled using an equation which relates the current price change to an integral, $\zeta(t)$ involving past price and price derivative history. Thus it is not only the current prices but the past prices and the past price derivatives that help in determining the future prices and predict the market behavior.

3.2 The Model

The price of the stock commodity is determined by the behavior of the share holders who might buy, sell or hold the stock. It is assumed that the share held by the investors is small compared to the total value in the market. An investor has four options (A,B,C,D). A- sell the stock, B- hold the stock, C- sell the cash, D- hold the cash. Each of these options of the investor defines the state in which he is. There may be several transfer of investors from one state to another just like the transfer of molecules from one energy state to another in the chemical kinetics - that is the laws of mass action. For example, transitions occur from B to A, from A to D, from D to C, and from C to B. The corresponding rate equations are then :

$$\frac{dA}{dt} = -k_1 A + k_4 B \tag{3.1}$$

$$\frac{dB}{dt} = k_3 C - k_4 B \tag{3.2}$$

$$\frac{dC}{dt} = -k_3 C + k_2 D \tag{3.3}$$

$$\frac{dC}{dt} = k_1 A - k_2 D \tag{3.4}$$

Where the k_i will depend on the history of the price and its derivative. Note that summing the four equations implies that A + B + C + D is constant.

The crucial assumptions on mass psychology of investors are embedded in the rate co-efficients k_i which have two factors. The first factor is the "emotional" aspect of the investor urges him to sell the stock when the prices are falling. It is assumed that each investor has some idea about the rise and fall of the stock in the past and the price history derivatives and remembers it through an exponentially failing memory. Faster the memory fails, more the emphasis on the current changes in price.We thus have a product of the fractional change in price, i.e., $P^{-1}\frac{dP}{dt}$, with a natural function for decay; the exponential, integrated over all past times. This yields the first integral of 3.5 below. The second factor is the "rational" aspect is based on the motivation to buy when the price is below the realistic or intrinsic value, $P_a(t)$ of the shock.Here, there is also a decaying exponential term due to intellectual inertia, or lag time between actual changes and investor action, multiplied by the fractional discount.

Hence we define the "investor sentiment" or tendency to buy as :

$$\zeta(t) = q_1 c_1 \int_{-\infty}^{t} e^{-c_1(t-\tau)} \frac{1}{P(\tau)} \frac{dP(\tau)}{d\tau} d\tau + q_2 c_2 \int_{-\infty}^{t} e^{-c_2(t-\tau)} [\frac{P_a(\tau) - P(\tau)}{P_a(\tau)}] d\tau \tag{3.5}$$

Here, q_1 is the contribution to the "emotional" aspect, q_2 is the introduction to the intellectual aspect, c_1^{-1} is the "memory length", and c_2^{-1} is the "intellectual inertia".e.g., a large q_1 and c_1 means means that a recent sharp price increase will improve investor sentiment. Choosing an appropriate monotone positive function of ζ as for instance, $\frac{1}{2} + \frac{1}{2}tanh(\zeta(t))$, one may write:

$$k_2 = \alpha[\frac{1}{2} + \frac{1}{2}tanh(\zeta(t))] \tag{3.6}$$

$$k_4 = \beta[\frac{1}{2} - \frac{1}{2}tanh(\zeta(t))] \tag{3.7}$$

Note that since k_2 is is the rate at which investors decide to buy stock while k_4 is the rate at which they decide to sell stock, we have assumed the relation $\frac{k_2}{\alpha} + \frac{k_4}{\beta} = 1$, where α and β are constant amplitudes .Due to simplifications, it is not necessary to derive k_1 and k_3.

Next, we discuss the equation for the price, P. The relative price change should be given by a function, f, of buyers and sellers. That is:

$$\frac{dP}{dt} = Pf(C/A) \tag{3.8}$$

where f is an increasing function such that f(1) = 0. For example, f(z) = $\delta log(x)$ is acceptable.

The assumptions inherent in 3.5, 3.6, and 3.7 lead to a preliminary interpretation for price fluctuations. Further the concept of ”panic buying and selling” has been modeled using the kinetic theory. The price fluctuations are a result of a difference between the perceived and the expected prices. The emotional tendency of the stock holder tends to serve as a destabilizing factor for this gap rather than bridging it. Suppose there is a price fluctuation with the price increasing a bit, due to behavior of stock holders $zeta(t)$ further increases and the same analogy can be applied to a slight decrease in price. On the other hand, the latter part of 3.5 is a stabilizing factor in that a deviation from actual value ($P_a(t)$) results in a change in investor sentiment which is in the direction of restoring equilibrium. This process is usually called “bargain hunting” or “profit taking”. Consequently, there is a basic competition between these stabilizing and destabilizing forces. Within our model, the result of this competition decides whether one will observe abrupt reversals in price (and fraction of buyers) in the absence of dramatic changes in fundamental value of the underlying securities.

3.3 Simplification of The Model

The equations 3.1, 3.2, 3.3, and 3.4 can be reduced to a single equation with the additional assumption that the transition from seller to sitter is very rapid. This is realistic since buy and sell orders are usually executed almost immediately. More precisely, we assume that :

$$k_1 = \frac{\bar{k_1}}{\epsilon} \tag{3.9}$$

$$k_3 = \frac{\bar{k_3}}{\epsilon} \tag{3.10}$$

Where, 0≤ $\epsilon \ll 1$ and $\bar{k_1}$ and $\bar{k_3}$ are O(1). Then letting $\epsilon \longrightarrow 0$, we see that A $\longrightarrow$0, C$\longrightarrow$0, k_1A $\longrightarrow$ k_4B, and k_3C$\longrightarrow$ k_2D, so that 3.1 reduces to :

$$\frac{dD}{dt} = k_4B - k_2D = -\frac{dB}{dt} \tag{3.11}$$

From this, it is clear that D = 1 - B and therefore that:

$$\frac{dB}{dt} = k_2(1 - B) - k_4B \tag{3.12}$$

$$\frac{C}{A} = \frac{k_2\bar{k_1}}{k_4\bar{k_3}}\frac{1 - B}{B} \tag{3.13}$$

which reduces the price equation (3.8) to one only involving B, namely:

$$\frac{dP}{dt} = Pf(\frac{k_2\bar{k_1}}{k_4\bar{k_3}}\frac{1 - B}{B}) \tag{3.14}$$

In togetherness, the equations 3.12 and 3.14 coupled along with equation 3.5, 3.6, and 3.7 constitute the model based on these assumptions.

Unlike many mathematical models of markets, a key feature of our model is the absence of explicit probabilistic concepts. The central assumption to our model is the use of a realistic price value of stock ,$P_a(t)$.

Chapter 4

CONCLUSION

CONCLUSION FOR CHAPTER 3: We have therefore seen that the mathematical model based on kinetic theory that we have used not only models the price fluctuations based on the current prices but also takes into account the past prices and the price history derivatives. Due weight-age has been given to customer motivation and behavior. The model does not use any explicit probabilistic concept but revolves around the concept of assigning a realistic value to stocks which forms a central concept in the stock market.

Bibliography

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