Model Reduction for Nonlinear Conservative Law

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Background

Let $u(x,t) : [a,b] \times [0,T] \to \mathbb{R}$ solve

$$\frac{du}{dt} + \frac{d}{dx} f(u) = 0$$

where f depends on a parameter μ.

- along the *characteristic line* $x(t)$ defined by

  $$\frac{dx}{dt} = f'(u(x(0),0))$$

  $u(x(t),t)$ is constant.
- If two characteristic line crosses, a shock $x(t)$ is formed. It is determined by *RH condition*:

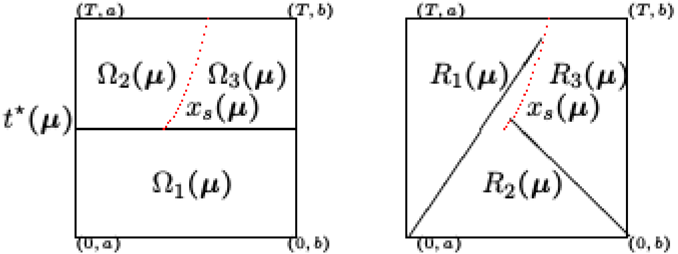
  $$\frac{dx}{dt} = \frac{f(u_-) - f(u_+)}{u_- - u_+}$$

Assumptions

- both sides $a \times [0, T]$, $b \times [0, T]$ are inflow boundaries.
- only one shock is formed in the region $[a, b] \times [0, T]$.
- continuous boundary conditions along $a \times [0, T]$, $b \times [0, T]$ and $[a, b] \times 0$.

Thus, a solution $u(x, t)$ can be be breaked down into three regular function, each defined on a region T_i such that $\cup T_i = [a, b] \times [0, T]$.

Partition



dotted line is the shock. Left: partition by the shock, right: partition by the characteristic line

Online Phase

- For $\mu \in \{\mu_1, \mu_2, \ldots \mu_n\}$ do:
- Lax-Friedrichs/Godunov monotone scheme to solve u
- Detect shock by maximum change in discrete first derivative, $(x(t), t_0)$
- Partition by the shock line to get three regular functions and store them in corresponding ROM model.

Offline Phase

- For Query μ do:
- polynomial interpolation to get t_0
- Use Newton's method and characteristic line to find u_- and u_+
- Use Rankine-Hugoniot condition to get $x(t)$
- Use ROM on each partitioned region to find interpolated functions.
- Output the resulting function, combined from three functions.

Remark

- Relies on assuming only a single shock line in $\mathbb{R}^2$. Generalize it to higher dimension is hard.
- Different interpolation on region where function is regular.