On the Identity of the Coefficient of Static Friction Between an Object and an Inclined Surface

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Let one consider a block of material standing stationary on a surface inclined at an angle θ. The forces acting on the object are of gravity F_g, the normal force F_N and the frictional force F_f. Given that $F_f = \mu F_N$ with μ being the coefficient of friction, specifically static if the object is stationary, denoted as μ_s, the coefficient of static friction may be found by knowing the angle θ of the inclined plane.

One may begin by stating that the net force with respect to the parallel component x may be given as

$$\sum F_{net_x} = F_{g_x} - F_f$$

and the net force with respect to the perpendicular component y as

$$\sum F_{net_y} = F_{g_y} - F_N$$

The force due to gravity is split up into its components F_{g_x} and F_{g_y} represented as

$$F_{g_x} = mg \sin\theta$$

$$F_{g_y} = mg \cos\theta$$

And the force of friction F_f is given as the product of the coefficient of static friction and the normal force F_N,

$$F_f = \mu_s F_N$$

Substituting these terms into the equation for $\sum F_{net}$, one has

$$\sum F_{net_x} = mg \sin\theta - \mu_s F_N$$

and

$$\sum F_{net_y} = mg \cos\theta - F_N$$

Because the object on the inclined plane is stationary, the net force can be set to 0 and thus

$$0 = mg\sin\theta - \mu_s F_N$$

$$0 = mg\cos\theta - F_N$$

Add terms with F_N to both sides,

$$\mu_s F_N = mg\sin\theta$$

$$F_N = mg\cos\theta$$

Divide the first equation by μ_s,

$$F_N = \frac{mg\sin\theta}{\mu_s}$$

$$F_N = mg\cos\theta$$

F_N can be substituted by the term $mg\cos\theta$ for the first equation as such,

$$mg\cos\theta = \frac{mg\sin\theta}{\mu_s}$$

Multiply both sides by μ_s,

$$\mu_s mg\cos\theta = mg\sin\theta$$

Divide both sides by $mg\cos\theta$,

$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta}$$

m and g cancel out, leaving

$$\mu_s = \frac{\sin\theta}{\cos\theta}$$

$\frac{\sin\theta}{\cos\theta}$ is equivalent to the tangent function $\tan\theta$, and therefore the equation simplifies to

$$\mu_s = \tan\theta$$

Therefore, the coefficient of static friction between the object and the inclined surface it rests on may be found by taking the tangent of the minimum angle of the inclined plane at which the object starts to accelerate.