Multi Numerica (Jan 2016)

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1 Geometric Power Series Function

Let $f(n,m,z,a) = n^a + n^{a+z} + ...n^{a+(m-1)z} = \sum_{i=0}^{m} n^{a+(i-1)z} = s$

$$= s = \sum_{i=0}^{m} n^{a+(i-1)z}$$

$$= sn^z = \sum_{i=0}^{m} n^{a+iz}$$

$$= sn^z - s = \sum_{i=0}^{m} n^{a+iz} - \sum_{i=0}^{m} n^{a+(i-1)z}$$

$$= sn^z - s = n^{a+mz} - n^a$$

$$= s(n^z - 1) = n^{mz+a} - n^a)$$

$$= s = \frac{n^{mz+a} - n^a}{n^z - 1}$$

$$\boxed{f(n,m,z,a) = \frac{n^{mz+a} - n^a}{n^z - 1}}$$

2 The Sum of the First n Hexagonal Numbers

$\sum_{i=1}^{n} \frac{(2i-1)(2i)}{2}$

$= \sum_{i=1}^{n} i(2i-1)$

$= \sum_{i=1}^{n} 2i^2 - i$

$= \sum_{i=1}^{n} 2i^2 - \sum_{i=1}^{n} i$

$= \frac{n(n+1)(2n+1)}{3} - \frac{n(n+1)}{2}$

$= \frac{2n(n+1)(2n+1)-3n(n+1)}{6}$

$= \frac{n(n+1)(4n+2-3)}{6}$

$= \frac{n(n+1)(4n-1)}{6}$

$$\boxed{\frac{n(n+1)(4n-1)}{6}}$$

3 The Sum of the First n s-gonal Numbers

Note that xth s-gonal number P(s,x) is

$P(s,x) = \frac{x^2(s-2)-x(s-4)}{2}$

Let $f(n,s) = \sum_{i=1}^{n} \frac{i^2(s-2)-i(s-4)}{2}$

$= f(n,s) = \sum_{i=1}^{n} \frac{i^2(s-2)}{2} - \frac{i(s-4)}{2}$

$$= f(n,s) = \tfrac{n(n+1)(2n+1)}{6}\tfrac{s-2}{2} - \tfrac{n(n+1)}{2}\tfrac{s-4}{2}$$

$$= f(n,s) = \tfrac{n(n+1)(2n+1)(s-2)}{12} - \tfrac{n(n+1)(s-4)}{4}$$

$$= f(n,s) = \tfrac{n(n+1)(2n+1)(s-2)-3n(n+1)(s-4)}{12}$$

$$= f(n,s) = \tfrac{n(n+1)((2n+1)(s-2)-3(s-4))}{12}$$

$$\boxed{f(n,s) = \frac{n(n+1)((2n+1)(s-2)-3(s-4))}{12}}$$

4 The n Products of the Sum of Squares

$$\prod_{j=1}^{n}(\sum_{i=1}^{j} i^2)$$

$$= \prod_{j=1}^{n}\tfrac{j(j+1)(2j+1)}{6}$$

$$= \tfrac{\prod_{j=1}^{n} j(j+1)(2j+1)}{6^n}$$

$$= \tfrac{n!(n+1)!\prod_{j=1}^{n}(2j+1)}{6^n}$$

$$= \tfrac{n!(n+1)!\frac{(2n+1)!}{2}}{2^{n-1}n!6^n}$$

$$= \tfrac{n!(n+1)!(2n+1)!}{2^n n!6^n}$$

$$= \tfrac{(n+1)!(2n+1)!}{12^n}$$

$$\boxed{\frac{(n+1)!(2n+1)!}{12^n}}$$