MA 240-02: Theory of Proof
Homework 1

NAME:

Instructions: Prepare neatly written and/or typed solutions to the following problems. Show all work, using complete sentences and proper notation/terminology. Be sure to explain your answers. You are welcome to discuss the problems with your classmates or with me, though everyone will need to submit their own solutions to the problems.

Problems denoted with a "**(L)**" must be typeset using LaTeX.

This homework set is due to my office (S-145H) by 4:00pm on Friday, September 13. If you use LaTeX, you may email me the pdf output prior to the 4:00pm deadline or hand in a hard copy. Any handwritten solutions must be submitted in hard copy form.

1. Determine which of the following are statements. For each statement, indicate the truth value. For each non-statement, explain why it is not a statement.

   (a) $x^3 - 8 = 0$

   (b) It is not the case that π is not a rational number.

   (c) $3x + 11$ is an even integer.

   (d) For every integer x, $3x + 11$ is an even integer.

   (e) This sentence is false.

2. Consider the following statements:

   $$P: \sqrt{2} \text{ is rational.} \qquad Q: 17 \text{ is even.}$$

   Write each of the following conditional statements in words and determine their truth values. Explain your reasoning!

   (a) $P \to Q$ (b) $Q \to P$ (c) $P \to P$ (d) $Q \to Q$

3. Let P be the statement "Student X passed every assignment in Calculus I" and let Q be the statement "Student X received a grade of C or better in Calculus I."

   (a) Suppose that Student X passed every assignment in Calculus I and received a grade of B, and that the instructor made the statement $P \to Q$. Would you say the instructor lied or told the truth? Explain your reasoning.

   (b) Suppose that Student X passed every assignment in Calculus I and received a grade of D, and that the instructor made the statement $P \to Q$. Would you say

the instructor lied or told the truth? Explain your reasoning.

(c) Suppose that Student X did not pass two assignments in Calculus I and received a grade of D, and that the instructor made the statement $P \to Q$. Would you say the instructor lied or told the truth? Explain your reasoning.

(d) Explain how parts (b), (c), and (d) are related to the truth table of $P \to Q$. Explain your reasoning.

4. **(L)** Write a complete proof of the following statement: If m is an odd integer, then $5m + 7$ is an even integer.

5. **(L)** Write a complete proof of the following statement: If m is an even integer, then $3m^2 + 2m + 3$ is an odd integer.

6. **(L)** Consider the following definitions: An integer a is said to be a **type 0 integer** if there exists an integer k such that $a = 3k$, a **type 1 integer** if there exists an integer k such that $a = 3k + 1$, and a **type 2 integer** if there exists an integer k such that $a = 3k + 2$. For example, 28 is a type 1 integer because $28 = 3(9) + 1$ and 9 is an integer; -7 is a type 2 integer because $-7 = 3(-3) + 2$ and -3 is an integer.

Use the above definitions to prove the following: If a is a type 1 integer and b is a type 2 integer, then $a + b$ is a type 0 integer.