Event

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Experiment

Definition (Experiment)

An experiment is the process by which an observation (or measurement) is obtained.

Example

- Roll a die
- Measuring the relative humidity of air
- Tossing a coin and observing the face that appears

A performance of an experiment is called a **trial** of experiment, and an observed result is called an **outcome**.
Note. One and only one of the possible outcomes will occur on any given trial of the experiment.

Sample Space

Definition (Sample Space)

The set of all possible outcomes of an experiment is called **sample space**, denoted by S.

Example

- Roll a die: S=$\{1, 2, 3, 4, 5, 6\}$
- Measuring the relative humidity of air: S=[0, 100]
- Tossing a coin and observing the face that appears: S=$\{$Head, Tail$\}$

Event

Definition (Event)

An event is any set of some possible outcomes of an experiment, that is, any subset of S.

An event is usually denoted by a capital letter, such as A, B, ...
We say that event A "occur" if the outcome of a trial lies in the event.

Sample space corresponds to an event that is certain to occur, and it is called a **certain event** or **sure event**. The empty set $\emptyset$ corresponds to an event that is impossible to occur, and it is called **impossible event**.

Example

Rolling a die and recording the number on the top,
A=$\{$the number is less than 7$\}$ is a certain event.
B=$\{$the number is greater than 7$\}$ is a impossible event;

Example 1: Rolling two dices

Example

Consider the game *The Settler of Catan*.

In this game, one player rolls two dices (to avoiding confusing, let them be one red and one blue) each turn, then all player can get some resources or nothing based on the numbers on the top of the dices.

Figure: The Settler of Catan

The rule is:
if the sum of the two numbers equals the number on the tile, then the adjacent player get one resource of the tile.

We see that if the sum is 3, 8 or 4, then
the red player can get one resource; if the sum is 4 or 11, the yellow player can get one resource.

Example 1 continue.

Example

The experiment is: rolling two different dices, and recording the numbers on the top.

The sample space is:

$$S = \left\{ \begin{array}{llllll} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3), & (4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

Example 1 continue.

Example

1. The event A is "the red player can get one resource". Then

$$A = \left\{ \begin{array}{cccccc} & (1,2), & (1,3), & & & \\ (2,1), & (2,2), & & & & (2,6), \\ (3,1), & & & & (3,5), & \\ & & & (4,4), & & \\ & & (5,3), & & & \\ & (6,2) & & & & \end{array} \right\}$$

Example 1 continue.

Example

2. The event B is "the yellow player can get one resource".

Then

$$B = \left\{ (1,3),\ (2,2),\ (3,1),\ (5,6),\ (6,5) \right\}$$

Example 1 continue.

Example

3. The event C is "both players can get one resource".
Then

$$C = \left\{ \begin{array}{ccc} & & (1,3), \\ & (2,2), & \\ (3,1) & & \\ & & \\ & & \end{array} \right\}$$

C is called the **intersect** of A and B, denoted by $C = A \cap B$, which means "both A and B occur".

Example 1 continue.

Example

4. The event D is "at least one player can get one resource". Then

$$D = \left\{ \begin{array}{cccccc} & (1,2), & (1,3), & & & \\ (2,1), & (2,2), & & & & (2,6), \\ (3,1), & & & & (3,5), & \\ & & & (4,4), & & \\ & & (5,3), & & & (5,6), \\ & (6,2), & & & (6,5) & \end{array} \right\}$$

D is called the **union** of A and B, denoted by $C = A \cup B$, which means "A, or B, or both A and B occur".

Example 1 continue.

Example

5. The event E is "The yellow player can get one resource but the red player cannot".

Then

$$E = \left\{ \begin{array}{ll} & \\ & \\ & (5,6), \\ (6,5) & \end{array} \right\}$$

E is called the **difference** of A and B, denoted by $C = A \backslash B$, which means "A occurs and B does not occur".

Example 1 continue.

- Any physical event can associate a subset of S, and any subset of S can associate a meaningful event;
- Intersect, union and difference are three basic operations on events. All the events in one operation should be contained in the same sample space.

Example 2: Toss coin(s)

If the sample spaces are different...

Assuming $A = \{\text{"at least one head"}\}$

Situation 1
Experiment:
Toss one coin

Sample space:
$S_1 = \{\text{H, T}\}$

Event:
$A_1 = \{\text{H}\}$

Situation 2
Experiment:
Toss two coins

Sample space:
$S_2 = \{\text{HH, HT, TT}\}$

Event:
$A_2 = \{\text{HH, HT}\}$

Situation 3
Experiment:
Toss three coins

Sample space:
$S_3 = \{\text{HHH, HHT, HTT, TTT}\}$

Event:
$A_3 = \{\text{HHH, HHT, HTT}\}$

$\cdots$

Logic of Probability

$$\underbrace{\text{Experiment} \longrightarrow \text{Sample space}}_{\textit{condition}} \longrightarrow \underbrace{\text{Event}}_{\textit{problem}} \longrightarrow \underbrace{\text{Probability}}_{\textit{result}}$$

Terms

- Experiment
- Sample space
- Event

In next Section, we will talk about

- Mutual Exclusive Events and All Inclusive Events
- Independent Events and Dependent Events
- Probability of an Event

Thanks