

# Dynamic Models and Their Applications: Paper assignment

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## I. INTRODUCTION

The relationship between unemployment and inflation remains as an important research question on theoretic and empirical macroeconomics. Does this relationship remains valid? Did it change over time? How can present values of unemployment help on predicting future inflation? Stock and Watson, in the paper *Forecasting Inflation* of 1999, are particularly interested in how the Phillips curve could be used to forecast inflation. They also investigated whether it was the best predictor or not.

The authors made a distinction between the traditional Phillips curve (using the unemployment rate and the NAIRU<sup>1</sup>) and alternative specifications using different economic activity indicators such as capacity utilization, personal income or industrial production. In addition, they use indicators of money supply, stock prices, output, wages, interest rates and exchange rates in order to compare the performance of these other variables and the Phillips curve on predicting inflation. They use monthly data from 1959 until 1997 of the United States, including two different measures of inflation: the Consumer Price Index (CPI) and the Personal Consumption Expenditure (PCE). The reason, why the authors decided to use different inflation indexes is the fact, that these measures differed from each other in some specific periods. As they were interested on forecasting, each estimation uses past data to predict the inflation of the 12 subsequent months (out-of-sample forecasting). Using the data from 1959 to 1969, the forecast starts from January 1970 and is done again every month until 1996, always using previous data.

As Stock and Watson (1999), we are also interested on the relationship of unemployment and inflation. Our analysis differs in terms of the data period, which starts just before the end of their sample. We use American monthly data covering the period from 1990 until 2013, for both the CPI and PCE. Differently from them, we are going to focus our analysis only on comparing the performance of different Phillips curve specifications on forecast. We are interested in checking whether other specifications of the Phillips curve (using other variables of aggregate activity) can over-perform the traditional specification using unemployment. We redo the estimation of the paper by 6 alternative indicators, namely: (1) industrial production (IP), (2) real personal income (GMPYQ), (3) total real manufacturing and trade sales (MSMTQ), (4) the number of employees on non-agricultural payrolls (LPNG), (5) the capacity utilization rate in manufacturing (IPXMCA), (6) and housing starts (HSBP).

First, we perform the estimation as the authors did: we forecast the 12 months inflation using the same out-of-sample methodology, redoing the forecast for each month, from 2002 until 2013. The first observation used in the regression is 1990:1, while the period over which simulated

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<sup>1</sup>Non-Accelerating Inflation Rate of Unemployment

out of sample forecasts are computed and compared is 2002:1-2013:12. Then we compare the results of the alternative specifications to the benchmark Phillips Curve. On the next section, we present which models were used to forecast and how we can compare them. Section 3 discusses the models. Section 4 compares our results with forecasts using simulated data. Finally, section 5 concludes the paper with a comparison with our results and the ones of Stock and Watson (1999), including possible explanations for the differences found.

## II. MODEL DEVELOPMENT

In developing the model, we used the same specification as Stock and Watson (1999) [1]. The main investigation of the paper is to compare different specifications of the Phillips curve, in particular, with varying indicators of aggregate economic activity. The authors compare between three specifications of the model:

- The traditional Phillips curve with unemployment rate as explanatory variable
- An alternative specification with different aggregate economic activities (industrial production, real personal income, capacity utilization rate in manufacturing etc.)
- Specification with different macroeconomic variables, such as money supply, stock prices, output, wages, interest rates and exchange rates

We take the former two specifications mentioned above, the traditional Phillips Curve and an alternative model with different indicators for economic activity. The first model specified in equation (1), *lhur* is the traditional Phillips curve with unemployment rate and its lags as variables for indicating aggregate economic activity. Notable feature in this particular model is that the dependent variable is change in inflation rate over periods longer than the sampling frequency (in this case, monthly). It allows predicting inflation change  $h$  periods ahead.

$$\pi_{t+h}^h - \pi_t = \phi + \beta(L)u_t + \gamma(L)\Delta\pi_t + e_{t+h} \quad (1)$$

This specification restricts the model in two ways. *First*, the inflation is I(1) process. Equation (1) is the same if we leave  $\pi_{t+h}^h$  on the left hand side and replace  $\gamma(L)\Delta\pi_t$  with  $\mu(L)\pi_t$  on the right hand side, with restriction  $\mu(1) = 1$  (the first restriction). For  $h = 12$ , this specification can be thought of as predicting inflation over the next twelve months using a distributed lag of current and past inflation, subject to the restriction that lag coefficients sum up to one. *Second*, the non-accelerating inflation rate of unemployment (NAIRU) is constant. Expressing the constant as  $\phi = -\beta(1)\bar{u}$  in Equation (3), we can see the second restriction imposed on the model.

$$\pi_{t+h}^h - \pi_t = \phi + \beta(L)(u_t - \bar{u}) + \gamma(L)\Delta\pi_t + e_{t+h} \quad (2)$$

The next model we treat, uses alternative macroeconomic indicators  $x_t$ s instead of  $u_t$  in equation (1). For variable  $x_t$  we took the six aggregate indicators mentioned above: IP, GMPYQ, MSMTQ, LPNG, IPXMCA and HSBP.

The IP, GMPYQ, MSMTQ, LPNAG are non-stationary I(1), thus we made transformations using the Hodrick-Prescott (HP) filter. Therefore, all  $x_t$  are treated as I(0). Specifically, we took the gap estimates from HP filter which is separated from the trend component. The transformation of the variables can be seen in the appendix. We chose the lag lengths of independent variables that minimizes the Schwarz information criterion (BIC), setting the maximum number of lags at 11 as it was proposed in the paper.

$$\pi_{t+h}^h - \pi_t = \beta(L)x_t + \gamma(L)\Delta\pi_t + e_{t+h} \quad (3)$$

In total, we estimate the model with seven different variables and its first differences for both price indexes, CPI and PCE.

### III. MODEL DISCUSSION

The theory about the relation between unemployment and inflation dates back to 1958. Phillips' empirical work showed a relation between unemployment rate and the rate of change on wages in the United Kingdom. Later on, Milton Friedman established a more modern specification for the relationship, introducing the natural rate of unemployment and stating that variations from this rate would impact changes on price inflation only on the short-run. The basic idea is that a higher economic activity (lower unemployment) indicates that the economy is producing closer to its maximum and, therefore, demand might exceed the supply capacity, raising prices.

As explained above, the main goal is to test various alternative specifications of the Phillips curve against the traditional model that uses unemployment. We must point out that each regression is estimated again every month and the forecast of the subsequent month is computed. In order to compare which forecast performed better, two measures are used. The first one is the *relative mean squared error (RMSE)*. The mean squared errors are calculated as follows: first we find the difference between real values of the inflation rate change and the forecast values. We square these differences and take the mean value of the squared errors. Then the relative mean squared errors are calculated as the ratio of MSE from a model using  $x_t$  to the benchmark model (using unemployment rate -  $u_t$ ).

If the  $RMSE > 1$ , then, the traditional specification of the model with  $u_t$  is outperforming the alternative one in terms of efficiency. Or, in other words, unemployment predicts future inflation better than a given variable  $x_t$ . By a forecast combining regression (4), we can derive the other measure used to compare forecasts performances:

$$\pi_{t+h}^h - \pi_t = \lambda f_t^x + (1 - \lambda)f_t^u + e_{t+h} \quad (4)$$

Equation (4) is a regression of the actual value of inflation on the two different forecasts  $f_t^x$  and  $f_t^u$ , with  $x_t$  and  $u_t$  respectively.  $\lambda$  indicates how much each forecast estimation add to each other. If the estimated  $\hat{\lambda} > 1$ , the  $f_t^x$  is a better forecast, which means that the forecasts based on unemployment rate adds nothing to the forecasts based on other economic activity measures. On the other hand, when  $\hat{\lambda} = 0$ , forecast based on unemployment  $f_t^u$  is better than the forecasts based on the  $x_t$ . We get these measures for every specification of the model and presented them in Table 1 of Appendix. This is the main replication of the model in Stock and Watson (1999). Table 2 in their paper does exactly the same as ours, but using different periods (1970-1996).

By the results on Table 2, we can see that the traditional Phillips curve is not the best predictor of future inflation for the period from 2000 to 2013. In the gaps specification, the RMSE is bigger than 1 only for housing starts (HSBP). The capacity utilization rate in manufacturing (IPXMCA) is slightly bigger than 1 only for the model using CPI. The  $\lambda$  are also substantially larger than 0 and give the same qualitative result as the RMSE. For the first differences specification, the same is valid. Only the number of employees on non-agricultural payrolls (LPNG) under-performs unemployment on forecasting CPI inflation. All the other estimates are better for predicting future inflation.

The comparison with the univariate equation reveals that the Phillips curve is a poor predictor of inflation on the period 2000-2013. The RMSE of the univariate regression is much smaller than

the benchmark and  $\lambda$  is bigger than 1 for both inflation indexes. Moreover, the RSME of the univariate case is also the lowest among all. It means that past values of inflation alone predicts its future values better than when including covariates related to economic activity. This result is different from the result of Stock and Watson (1999) and it is probably a consequence of the period we used.

Between 2000 and 2013, it seems that unemployment and inflation were barely related. Figure 1 in the appendix shows the series of Unemployment, CPI and PCE from 1970 to 2013. Before 2000, it is possible to observe a negative relationship between the two variables on the short-run. However, after 2000, and especially after 2008 (economical crisis), the relation is almost absent. Figure 2 only focus on the period 2000-2013 and we can see better that, in 2008, there was a great increase on unemployment rate (from around 4% to 11%). The inflation rate measured by the CPI has fallen in 2008<sup>2</sup>, but it seems to behave independently of unemployment after the initial shock. For the case of PCE inflation, the drop was small in 2008, and it continued at similar levels even after unemployment started to fall again (unemployment rate was lower than 6% in 2013). This pattern explains why the RMSE of the univariate model using PCE inflation was so low (0.11). Overall, it shows that the forecast using the Phillips curve, as performed by Stock and Watson (1999), is not successful for predicting the inflation of recent years. Moreover, among those specifications, the one using unemployment was one of the worst for predicting future inflation.

#### IV. MODEL EVALUATION

In order to evaluate the model presented in the paper, we performed a number of steps. First, we decided to estimate Equation 3 in-sample for both CPI and PCE in period 1990:1-2013:12. Then we diagnosed the property of the residuals, namely independence. What we observe is that the residuals from the estimated model are strongly correlated over time, which of course violates the model assumption. The Q-statistics of the Ljung-Box tests as well as both autocorrelation and partial autocorrelation functions are presented in Figures 9 (for PCE) and 11 (for CPI) in the Appendix.

Second, using the parameters from the model, we simulated inflation using Monte Carlo procedure with 1000 repetitions. In our simulated data, errors are normally distributed and uncorrelated. The variance of the errors was adjusted using the following formula:  $\sigma_{sim}^2 = (\sigma_{\pi} - \sigma_{fit})^2$ , where  $\sigma_{\pi}$  is the standard deviation of the true inflation and  $\sigma_{fit}$  is the standard deviation of the fitted values from our model. In the Figures 10 and 12 in the Appendix, there are the PACF and ACF with Ljung-Box test statistics for simulated PCE and CPI respectively. We can clearly see that the simulated model meets the assumption. In addition, for all lags, we fail to reject the null hypothesis that residuals are independently distributed.

Based on our 1000 simulated samples, we estimated exactly the same models as using actual data and made a set of in-sample forecasts for the period 2002:1-2013:12. Next, for each Monte Carlo repetition, we calculated the mean-squared error of the simulated inflation forecast and them compared it to the mean-squared error of the forecasts based on real values of inflation. As a result we obtain 1000 relative MSE for both CPI and PCE. The results of this simulation procedure are presented in the Table 2 in the Appendix.

Due to the fact that in the model based on true data the assumption of dependency was violated, mean-squared errors of the forecasts were much larger than for our simulated data. We can also observe that the relative mean squared errors obtained from real CPI and simulated

<sup>2</sup>The CPI inflation puts much more weight on real state than the PCE one, which explains the strong fall right after the crisis (when house prices has fallen substantially), differently from the PCE

CPI are almost two times smaller to those obtained from PCE simulation procedure. This might suggest that the problem of errors autocorrelation is much larger for the PCE inflation index.

## V. CONCLUSIONS

Figure 1 plots annual inflation rates,  $\pi_t^{12}$  for two US monthly price indexes. The pattern of inflation calculated by these two indexes are same as the original paper between 1970 and 2000. We focus the analysis on inflation measures between 2000 and 2013 which are plotted in Figure 2. During this time window, these two measures have different patterns. First, CPI inflation varies more than PCE inflation does. Second, PCE inflation is larger than CPI inflation most of time except in year 2008. The reason, why the CPI is much more volatile than PCE in our sample, is the fact that in case of CPI the weight put on housing prices is substantially larger than in case of PCE (40% and 22% respectively).

The results of inflation forecasts based on measures of aggregate real activity are shown in Table 1. We forecast the inflation between 2000 and 2013 rather than divide it into two sub-periods as what the paper does and have the following findings:

First, same as the original paper, we also find that there are important differences in the forecastability of inflation across price measures. PCE inflation forecasts are more accurate than CPI forecasts. For the univariate case, the RMSE for PCE is only one fourth of that for CPI. For the other aggregate real activity factors, only two variables (MSMTQ and HSBP)'s RMSEs are larger than those for CPI. For the first differences specification, the RMSE for PCE is much smaller.

Second, most of the estimated values of  $\lambda$  are significantly greater than 0. In contrast to the magnitude of  $\lambda$  in the original paper, the estimated  $\lambda$  in our results are larger and most of them are larger than 1 and some even larger than 2. This suggests that these alternative activity measures contain useful information not included in lags of the unemployment rate or past inflation. Another difference about forecastability is that more than two variables outperform the unemployment rate uniformly across series. This is the case especially in the first differences specification. All variables except LPLNG outperform the unemployment rate.

Finally, in our case, specifications using the first difference of the activity variables and specifications using 'gaps' produce similar forecasts by comparing RMSE. For the CPI series, first difference specification produces a slightly larger RMSE but for PCE series, the gaps specification produces a slightly larger RMSE. However, such differences are quite small.

Based on the results in Table 1, we get the same conclusion as that paper that forecasts can be improved upon using a generalized Phillips curve based on measures of real aggregate activity other than unemployment. However, we in our simulation exercise we detected that the residuals from the models estimated for 1990:1-2013:12 are characterized by high level of autocorrelation, which violates the assumption of that the errors are independent.

## REFERENCES

- [1] James H. Stock, Mark W. Watson, *Forecasting inflation*, Journal of Monetary Economics 44 (1999) 293-335

## VI. APPENDIX

Table 1: Forecasting performance of alternative real activity measures

Forecasting period: 2000-2013		PUNEW (CPI)		GMDC (PCE)	
Variable	Trans	Rel. MSE	$\lambda$	Rel. MSE	$\lambda$
Univariate		0.53	1.02 (0.05)	0.14	1.03 (0.05)
GARCH(1,1)				0.11	0.91 (0.03)
<i>Gaps specification</i>					
ip	DT	0.94	0.77 (0.41)	0.79	0.97 (0.20)
gmpyq	DT	0.94	1.11 (0.55)	0.80	1.25 (0.33)
msmtq	DT	0.53	0.91 (0.19)	0.71	0.81 (0.14)
lplng	DT	0.77	1.21 (0.38)	0.74	1.00 (0.28)
ipxmca	LV	1.02	0.11 (0.73)	0.75	0.96 (0.20)
hsbp	LN	1.34	-0.16 (0.37)	1.38	0.09 (0.31)
<i>First differences specification</i>					
ip	DLN	0.92	1.24 (0.61)	0.76	1.43 (0.36)
gmpyq	DLN	0.95	1.33 (0.73)	0.76	1.49 (0.36)
msmtq	DLN	0.92	2.27 (0.70)	0.71	1.48 (0.37)
lplng	DLN	1.06	-0.13 (0.94)	0.79	1.80 (0.53)
ipxmca	DLV	0.89	1.64 (0.51)	0.74	1.38 (0.35)
hsbp	DLN	0.95	1.55 (0.85)	0.76	1.46 (0.35)
dlhur	DLV	0.96	1.24 (0.90)	0.75	1.61 (0.44)

In parentheses: HAC robust standard errors (estimated using a Barlett kernel with 12 lags) as used in the original paper.

Note: For a series  $y_t$ , the transformations  $x_t = f(y_t)$  are:  $x_t = y_t$  (LV),  $x_t = \Delta y_t$  (DLV),  $x_t = \Delta^2 y_t$  (DDLV),  $x_t = \ln(y_t)$  (LN),  $x_t = \Delta[\ln(y_t)]$  (DLN),  $x_t = \Delta^2[\ln(y_t)]$  (DDLN),  $x_t = \ln(y_t) - \tau_t$  (DT) where  $\tau_t$  is the HP-trend  $y_t$

Table 2: Relative mean-squared errors obtained from simulation Monte Carlo procedure (1000 repetitions)

	Min	Mean	Max	Std. dev.
PUNEW	3.31	4.07	7.64	0.59
GMDC	4.76	7.07	11.96	0.88

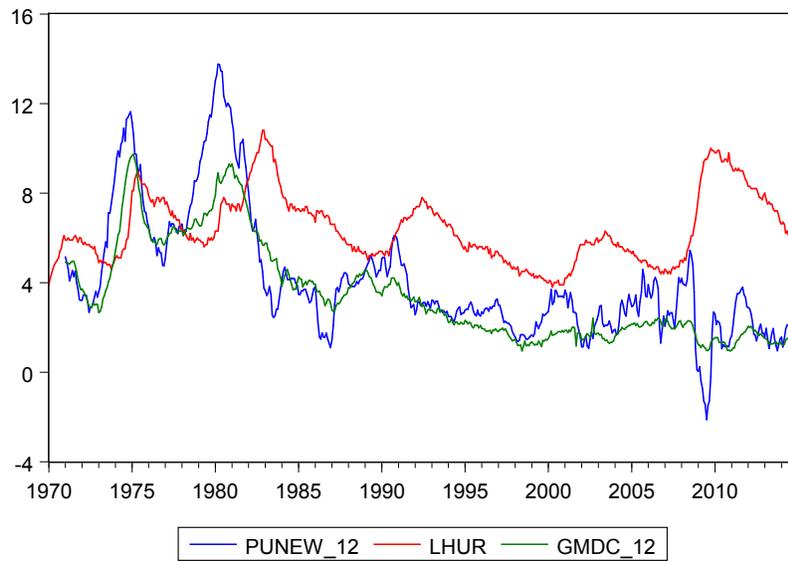


Figure 1: Unemployment and Annual inflation (CPI and PCE) 1970-2013

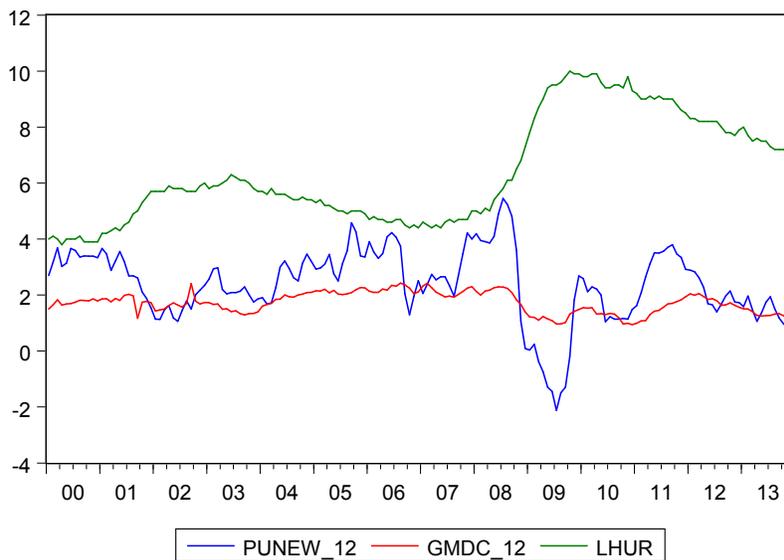


Figure 2: Unemployment and inflation 2000-2013 (CPI and PCE)

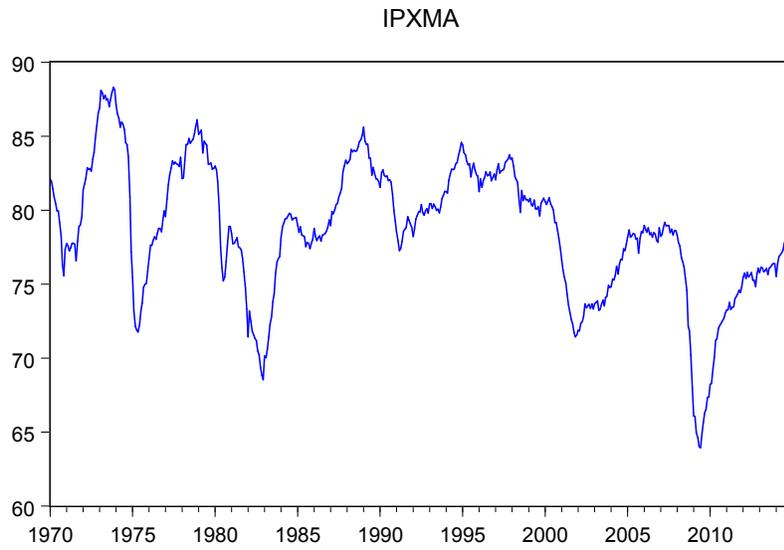


Figure 3: Capacity utilization in manufacture 1970:1-2014:12

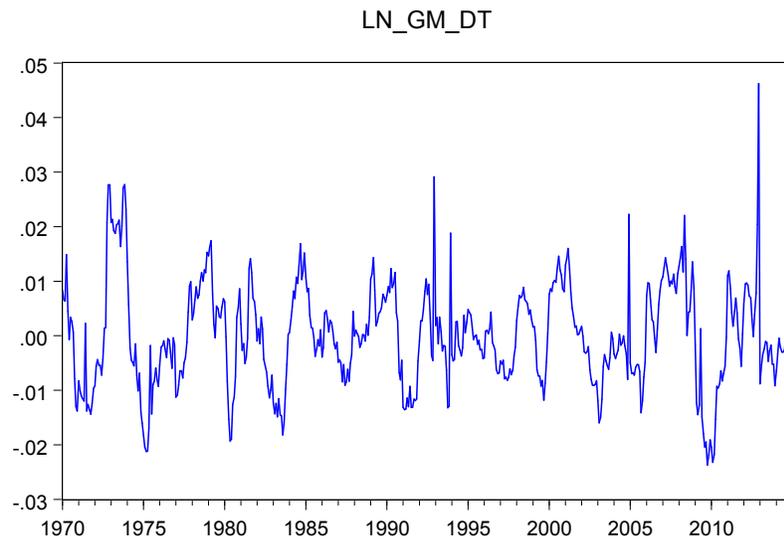


Figure 4: Real personal income (in logs, detrended) 1970:1-2014:12

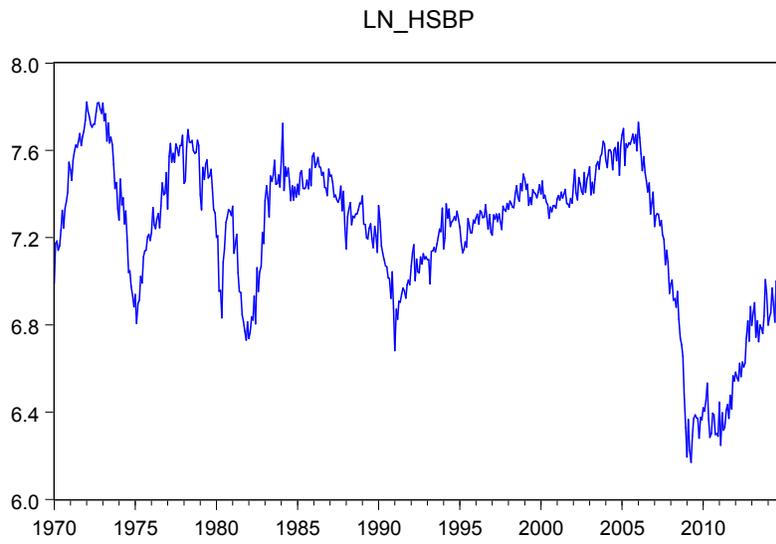


Figure 5: Housing starts (in logs) 1970:1-2014:12

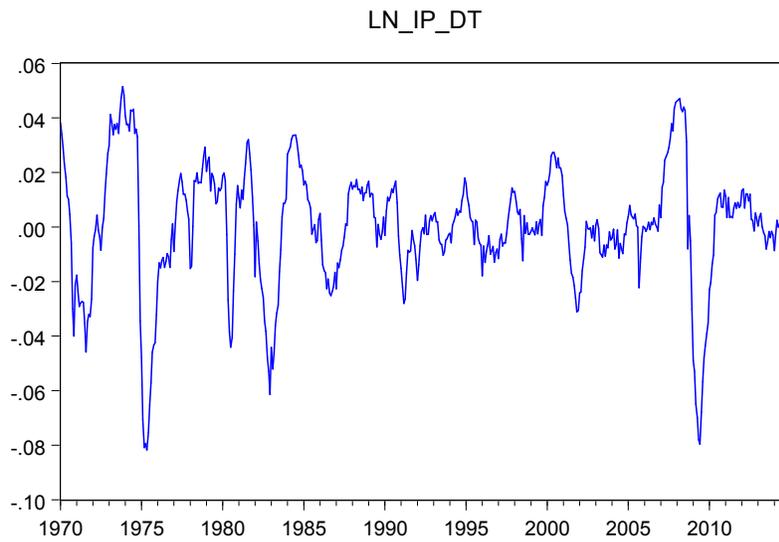


Figure 6: Industrial production (in logs, detrended) 1970:1-2014:12

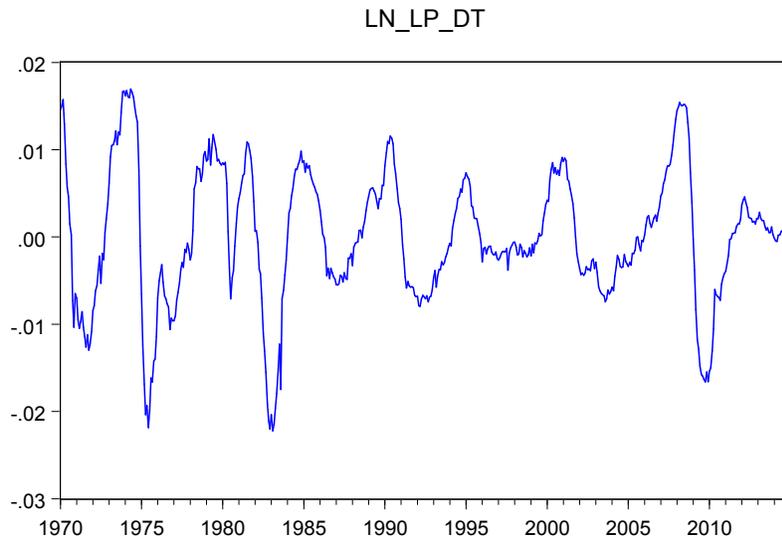


Figure 7: Non-agricultural payrolls (in logs, detrended) 1970:1-2014:12

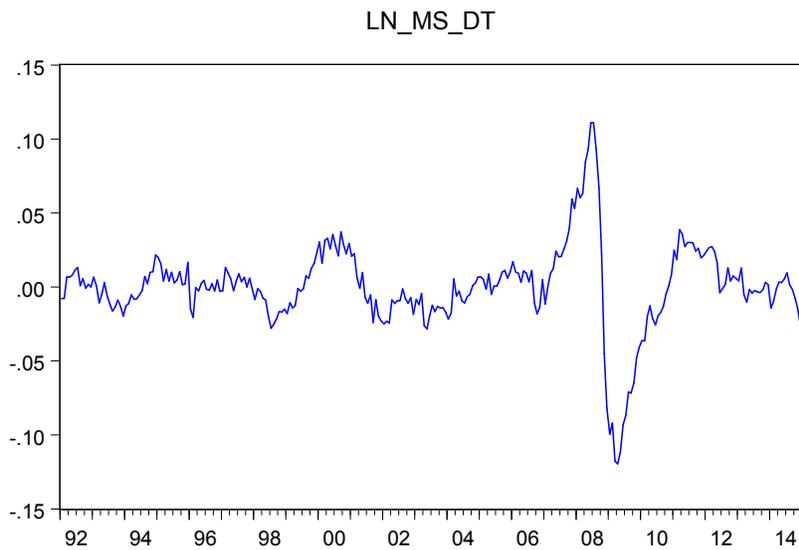


Figure 8: Total real manufacturing and trade sales (in logs, detrended) 1992:1-2014:12

Correlogram of Residuals

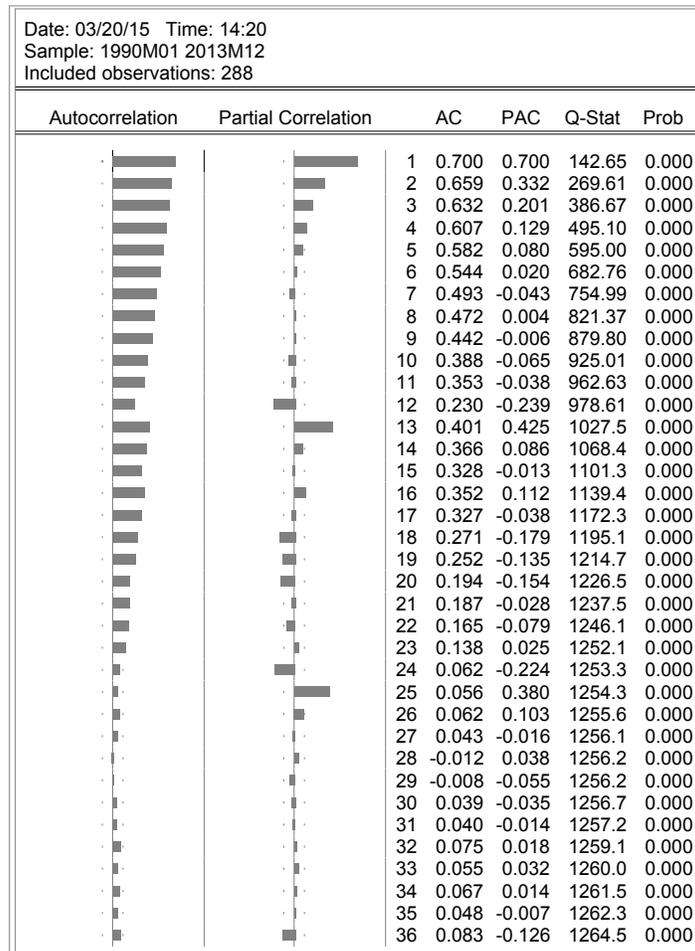


Figure 9: Correlogram of the residuals from Philips curve model of PCE inflation (1990:1-2013:12)

Correlogram of Residuals

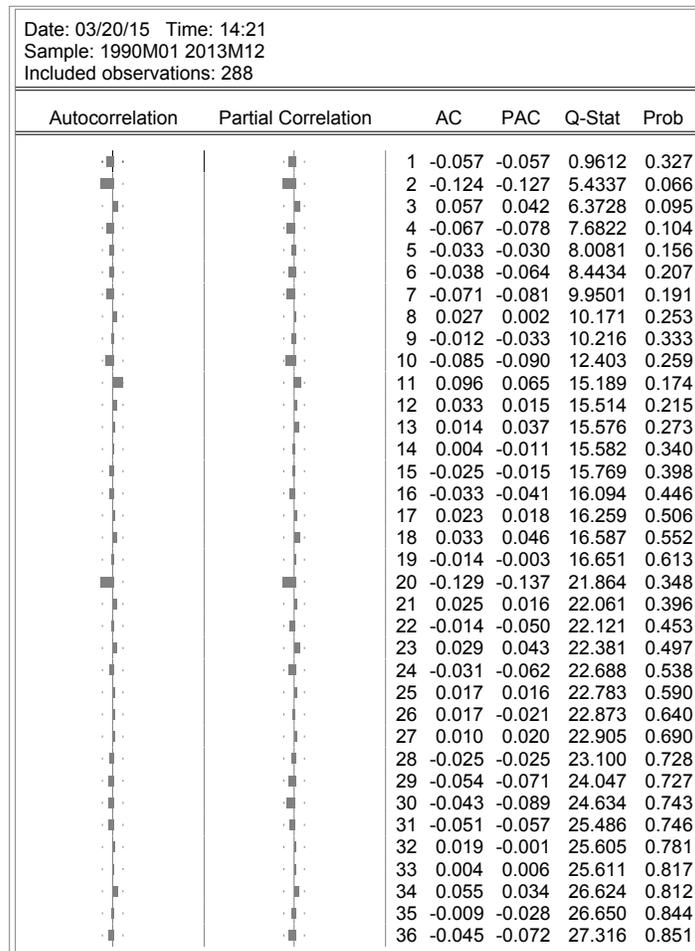


Figure 10: Correlogram of the residuals from Philips curve model of simulated PCE inflation (1990:1-2013:12)

Correlogram of Residuals

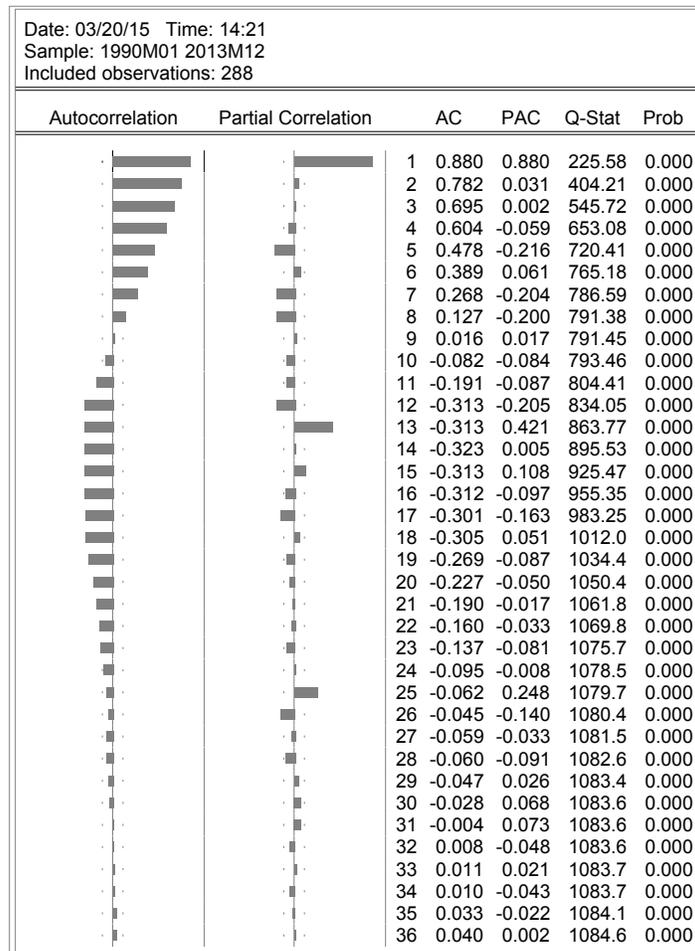


Figure 11: Correlogram of the residuals from Philips curve model of simulated CPI inflation (1990:1-2013:12)

Correlogram of Residuals

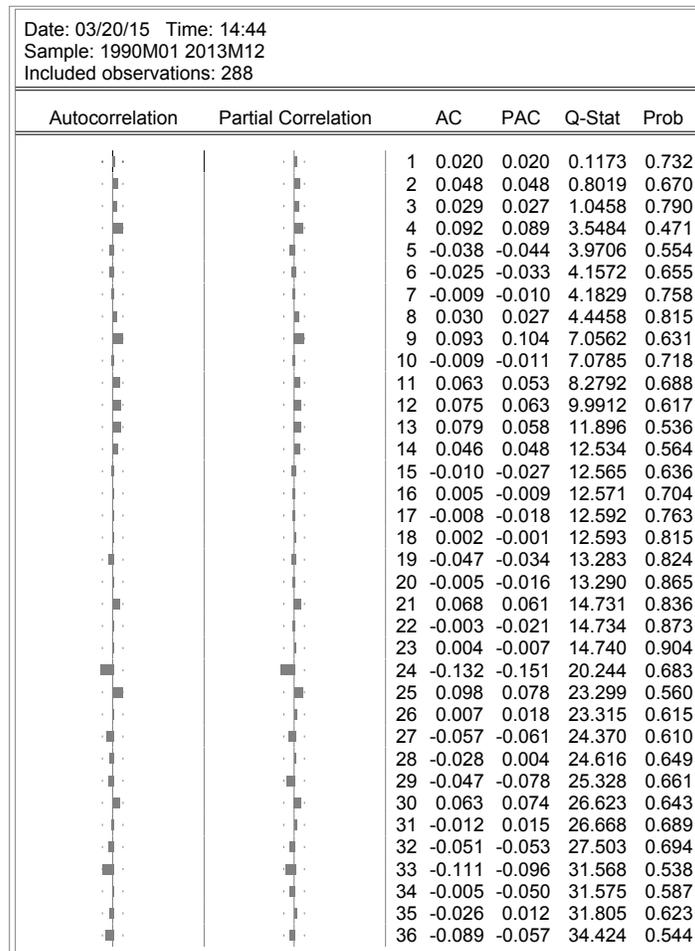


Figure 12: Correlogram of the residuals from Philips curve model of simulated CPI inflation (1990:1-2013:12)