# Analyzing elliptically polarized light using Fresnel Rhomb

Experiment-1

17-01-2017

**Abstract**

The aim of the experiment is to study elliptically polarized light using a Fresnel rhomb and to determine the ratio of semi major axis to the semi minor axis of the various elliptically polarised light forms for different angles of plane polarised incidence at the rhomb

## 1 Apparatus used:

1. He-Ne Laser source
2. Fresnel Rhomb
3. Polarising and analysing Polaroids
4. Photo-diode
5. Micro ammeter
6. Suitable optical bench with uprights to hold the accessories

## 2 Theory

### 2.1 Polarisation

Light wave is a transverse electromagnetic wave made up of mutually perpendicular, oscillating electric and magnetic fields perpendicular to each other.

When a beam of randomly polarised light is incident on the surface of an anisotropic crystal [1], it separates into two rays- the Ordinary Ray(o-ray) and the Extraordinary Ray(e-ray). They are linearly polarised in mutually perpendicular directions. This phenomenon is called Birefringence or Double Refraction.

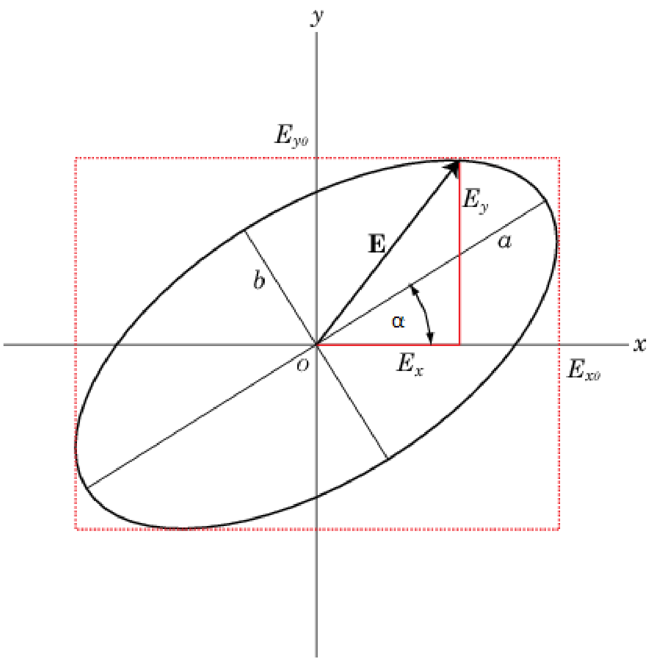
The e-ray and the o-ray undergo superposition after emerging from an anisotropic crystal. They combine to produce different states of polarisation depending upon their optical path difference:

[1] An anisotropic crystal is one in which properties like conductivity, velocity of light, refractive index etc. depend on the direction along which the property is measured

1. When the optical path difference is 0 or an even or odd multiple of $\lambda$ /2, the resultant light wave is linearly polarised.
2. When the optical path difference is $\lambda$ /4, and the wave amplitudes are equal, the resultant light wave is circularly polarised.
3. For any other path difference, the resultant light wave is elliptically polarised.

## 2.2 Elliptical Polarisation

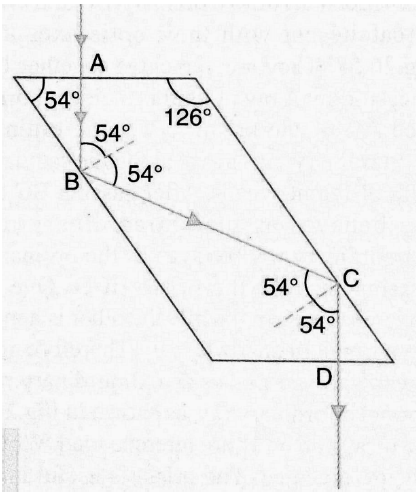
Elliptical polarization is the polarization of electromagnetic radiation such that the tip of the electric field vector describes an ellipse in any fixed plane intersecting, and normal to, the direction of propagation. An elliptically polarized wave may be resolved into two linearly polarized waves, with their polarization planes at right angles to each other.




At a fixed point in space (or for fixed z), the electric vector $\vec{E}$ traces out an ellipse in the x-y plane. The semi-major and semi-minor axes of the ellipse have lengths a and b. While the maximum amplitude of the $\vec{x}$ and $\vec{y}$ components of $\vec{E}$ are $E_{xo}$ and $E_{yo}$. The major axis of the ellipse is inclined at an angle $\alpha$ with respect to $\vec{x}$ .

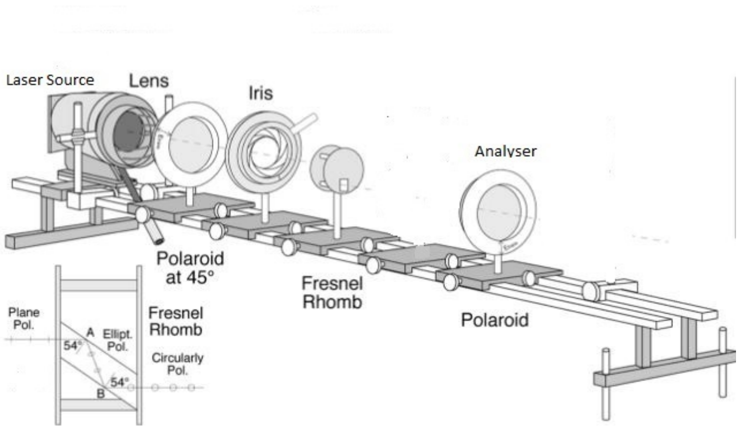
## 2.3 Fresnel Rhomb

A Fresnel Rhomb is a prism designed for converting linearly polarized light into elliptically polarized light. The device works based on the relative phase shift between the the two linearly polarized components of the incident light as a result of total internal reflection, two of which each contribute a nominal retardance [1] of $45^o$ . The rhomb has interior angles of $54^o$ and $126^o$.




## 2.4 Production of Elliptical Polarisation

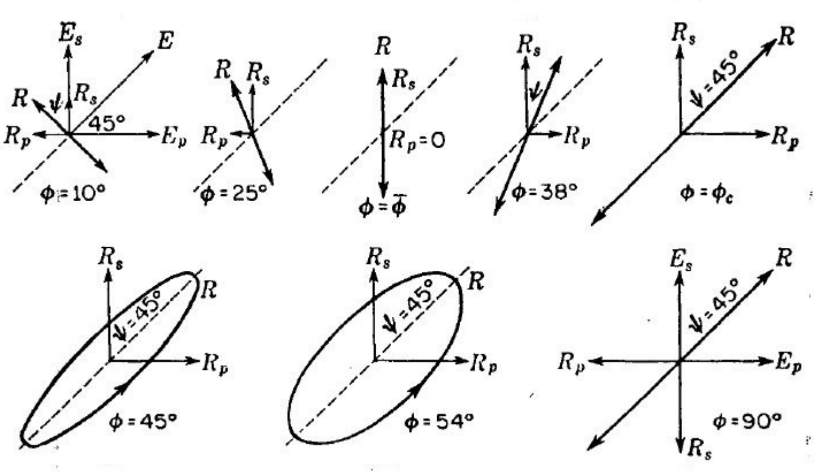
Elliptically polarised light can be produced when two mutually perpendicular plane polarised waves of unequal amplitude and any phase difference are superimposed.




As shown in the figure, laser is first passed through a polariser, the plane polarised light which comes out of the polariser is incident normally on the Fresnel rhomb in such a way that the incident light undergoes two internal reflections before transmitting perpendicularly on a parallel surface. The phase change for the internally reflected light happens to be a function of the angle of incidence. For the material used in Fresnel Rhomb (n=1.51), a phase change of $45^o$

[1]Nominal Retardance - The difference in phase shift between two characteristic polarizations of light upon reflection from an interface.

happens at two angles of incidences $48^o37'$ and $54^o37'$. Fresnel constructed the rhomb such that both the internal reflections occur at an angle of $54^o37'$ which contributes to a total phase change of $\pi/2$. Thus an incident linearly polarised light can be transmitted as an elliptically polarised light.




**Modes of light vibrations internally reflected in glass at various angles of incidence.**

When this elliptically polarised light is made to fall on the analyser, it filters the beam such that components of only a specific polarization is transmitted. When the analyser is rotated about the transmitted axis, intensity of the transmitted light changes from maximum to minimum, according to whether the principal plane of the analyser is parallel or perpendicular to the major axis of the elliptical polarization. The light transmitted from the analyser is made to fall on a photocell which converts the optical energy into electrical energy and the current through the photocell gives a measure of the intensity transmitted by the analyser.

Thus the variations in the intensity of the light for different orientations of the analyser can be studied fairly accurately with the help of the photocell.

## 2.5 Superposition of waves linearly polarised at right angles

Plane of incidence : x-z
Let the incident polarisation be inclined at an angle $\phi$ with the plane of incidence. Then,

$$\vec{E_{inc}} = E_o \cos\phi \cos\omega t \hat{i} + E_o \sin\phi \cos\omega t \hat{j} \tag{1}$$

On passing through the Fresnel rhomb, a phase difference of $\frac{\pi}{2}$ is introduced between the two components of the incident polarisation vector. Hence, the transmitted electric field vector can be written as

$$\vec{E_{trans}} = E_o \cos\phi \cos\omega t \hat{i} + E_o \sin\phi \sin\omega t \hat{j} \tag{2}$$

$$\vec{E_{trans}} = E_x \hat{i} + E_y \hat{j} \tag{3}$$

where,

$$\vec{E_x} = E_o \cos\phi \cos\omega t \tag{4}$$

$$\vec{E_y} = E_o \sin\phi \cos\omega t \tag{5}$$

Thus,

$$\frac{E_x}{E_o \cos\phi}^2 + \frac{E_y}{E_o \sin\phi}^2 = 1 \tag{6}$$

The above is the equation of an ellipse, of the form

$$\frac{x}{a}^2 + \frac{y}{b}^2 = 1 \tag{7}$$

where,

$$x = \frac{E_x}{E_o}, y = \frac{E_y}{E_o}, a = \cos\phi and b = \sin\phi \tag{8}$$

Place an analyser at an angle $\theta$ with respect to x. Where $\hat{n}$ represents the axis of the analyser.

$$\hat{n} = cos\theta\hat{i} + sin\theta\hat{j} \tag{9}$$

Component of transmitted Electric field is :

$$E_n = \vec{E_{transmitted}}.\hat{n} = E_0 cos\phi cos\theta coswt + E_0 sin\phi sin\theta sinwt \tag{10}$$

$$\begin{aligned} {E_n}^2 = {E_0}^2[&cos^2\phi cos^2\theta cos^2wt + sin^2\phi sin^2\theta sin^2wt \\ &+ 2cos\phi sin\phi sin\theta cos\theta coswtsinwt] \end{aligned} \tag{11}$$

$$I \propto < {E_n}^2 > \tag{12}$$

Intensity is proportional to the time average of the $E_n$

$$< {E_n}^2 >= \frac{{E_0}^2}{2}[cos^2\phi * cos^2\theta + sin^2\phi sin^2\theta] \tag{13}$$

$$< {E_n}^2 >= \frac{{E_0}^2}{2}[cos2\phi * cos^2\theta + sin^2\phi] \tag{14}$$

$$I = I_0[cos2\phi * cos^2\theta + sin^2\phi] \tag{15}$$

Varying $\theta$ over $360^o$ would produce a variation in intensity

$$I_{max} : \theta = 0, \pi \tag{16}$$

$$I_{max} = I_0[cos2\phi + sin^2\phi] \tag{17}$$

$$I_{max} = I_0[cos^2\phi - sin^2\phi + sin2\phi] \tag{18}$$

$$I_{max} = I_0 cos^2\phi \tag{19}$$

$$I_{min} : \theta = \pi/2, 3\pi/2 \tag{20}$$

$$I_{min} = I_0 sin^2\phi \tag{21}$$

$$\frac{I_{min}}{I_{max}} = tan^2\phi \tag{22}$$

Thus, knowing $I_{max}$ and $I_{min}$ we can find the polarizer angles for the respective observation sets.

# 3 Precautions

1. Care should be taken to see that no external light other than the light from the analyser enters the photo-diode.
2. The height of the uprights should be properly adjusted so that the light from the source reaches the photo-diode after passing through the polariser, rhomb and the analyser.
3. While taking observations, the analyser should be rotated in one direction only to avoid backlash error.

## 4 Procedure

1. Level the bed of the optical bench with the help of a spirit level and the screws provided. Switch on the laser source and adjust the height of the uprights so that the light from the source enters the polariser, rhomb, analyser and then the photo-diode.

2. Rotate the polariser and analyser to read zero on the respective circular scales. By rotating the analyser, see that the intensity of the transmitted light indicated by the micro ammeter reading varies between maximum and minimum on one full rotation, but is never zero.

3. Keeping the polariser at a fixed $\theta$, rotate the analyser from $0^o$ to $360^o$ in steps of $10^o$ and note the corresponding reading in the digital microammeter.

4. Now, rotate the polariser and note the readings in the digital microammeter.

5. Repeat the observations for different values of polariser angles.

## 5 Data analysis

1. Plot a polar graph between the orientation of the analyser and the corresponding microammeter reading for each orientation of the polariser.

## 6 Result

The elliptically polarized light from the Fresnel's Rhomb was studied for different polarizing angles and the difference between polarizing angles was found using the ratio of the minimum and maximum intensities was calculated.

## 7 Sources of Error

1. The Fresnel Rhomb used in the experiment section had a narrow range over which a transmitted ray underwent two internal reflections, when the light beam was incident normal to the plane. Thus, there was a huge loss in the maximum intensity derivable from this set-up because the rhomb was not able to accommodate the entire laser cross section. Thus the laser was incident at an angle to the normal of the plane in a way that it was able to transmit maximum intensity. This would lead to an error, as it deviates from the analysis employed which requires normal incidence.

2. The ability to accurately measure the rotation of the analyzer. The analyzer could slip back and forth in its holder such that even when a certain value's tick mark was lined up with the reference mark the actual tilt could be several degrees different.

3. The error due to improper alignment of the apparatus.